The TREND IN ENGINEERING

at the University of Washington

In this issue:

A MODEL STUDY OF SOME FISH FACILITIES FOR THE PROPOSED MAYFIELD DAM
Part II: Upstream Migrant System

THE APPLICATION OF DIGITAL COMPUTERS IN POWER-SYSTEM HYDRO-STORAGE PROBLEMS

APPROXIMATE LAMINAR SKIN FRICTION IN COMPRESSIBLE FLOWS WITH FAVORABLE PRESSURE GRADIENTS

NOUN PHRASE ANALYSIS USING THE IBM 650 COMPUTER

ZONE MELTING: A PHYSICAL METHOD FOR CONTROLLING IMPURITIES IN METALS

AN AID IN THE PRELIMINARY DESIGN OF PRESTRESSED CONCRETE BEAMS

April, 1957

Published quarterly

by

THE ENGINEERING EXPERIMENT STATION

RECOGNIZING AND DEVELOPING CREATIVE POTENTIAL*

JOHN E. ARNOLD

Professor of Mechanical Engineering, Massachusetts Institute of Technology

YOU CANNOT TELL, by examining the end product, whether it was obtained by means of creative activity or by means of more prosaic, routine methods, or even by pure chance. And, you cannot tell, by looking at the types of problems that men set for themselves, whether they are creative or noncreative. The great scientists of the past and the present are great because they attacked their analytical problem with great daring and imagination.... Unfortunately, it is also possible to arrive at satisfactory answers to the multisolutional, creative type of problem by extremely prosaic, nonimaginative, noncreative means....

Attributes of the Creative Individual: Now, what are some of the attributes that define or describe the creative individual? In general, they fall into two categories—mental and emotional. The creative person is more "sensitive to problems" than the noncreative person. He can see need areas that require attention long before the less sensitive person. He not only has the ability, but he enjoys searching in the dark areas and leaves the work in the well-lighted places to his less imaginative brethren. He has developed a constantly questioning attitude, a healthy skepticism, rather than a complacent acceptance. The questions he asks and the problems he poses for himself are stated in such a way that they do not limit or confine his thinking or actions. Correct problem statement is basic to successful creative activity. Incorrect problem statement can precondition the thinking and preclude desirable solutions. It also can have a tremendously inhibiting effect on three other mental attributes: fluency, flexibility and originality.

Fluency of Thought: The creative person is more fluent in his thinking than the noncreative person. He has more ideas per unit time than others. Obviously, they are not all good ideas. But when he completes his list of alternatives, the probability is very high that there will be one or more ideas that are novel and that will solve his problem better. While it is very doubtful that quantity breeds quality, my experience has been that those who have lots of ideas, usually have the best ones.

Flexibility of Thought: Flexibility of thinking reflects itself in the wide variety of approaches that the creative person chooses to investigate. The noncreative person's past experience provides him with a comfortable little rut in which to operate, and he has great difficulty getting out of that groove.

One of the worst examples of rigidity of thinking that I see as I go around and talk to and work with various industrial groups is the way in which they limit the design and research activity to only one or two of the four broad areas in which design activity can take place. Products or machines can be improved by increasing their function, by making the item do more than it did before, or by making it accomplish its prime purpose by an entirely new approach. The performance level of a product can be raised by making the product longer lived, more reliable, more accurate, safer and more convenient to use, and easier to repair and maintain. The cost may be lowered by eliminating excess parts, substitution of cheaper materials, cheaper and more efficient manufacturing methods, designing for convenience of assembly, designing to reduce hand labor or for complete automation. Last, products can be improved by designing for increased salability, that is, improving the appearance by making the product, its package and its point of sale more attractive, and by having a better appreciation of public wants.

Originality: The creative person thinks of more unusual, more unique solutions to problems than the noncreative person. He apparently has the ability to penetrate very deeply to remote regions. He can break down the seemingly natural boundaries within his own mind, so that his experience, instead of being pigeon-holed, is completely and carefully cross referenced. The creative person, then, is more sensitive to problems, he is more fluent and flexible in his thinking, and he is more original in the solutions he arrives at. We might say, he is better able to question, observe, associate and predict.

Emotional Attributes: The mental attributes listed are necessary, but not sufficient to the creative personality. Along with them there are a number of emotional attributes that many, I am sure, would list as primary rather than secondary. First of all, a man must be motivated before he will begin to attempt to solve any kind of problem, analytical or creative. We have ample evidence that over-motivation is extremely disastrous, in that it inhibits creative activity, but I am afraid that more of us suffer from under-motivation rather than over-motivation. Passivity is probably the original sin, and initiative is what most of us lack, for once into a problem, most of us can generate enough interest to keep us going until the problem is solved.

Willingness to take a chance, to gamble, may be another form of initiative. But it is an important aspect of the creative person's personality. The creative person has to be daring. He has to be a leader in his group for society, and he must constantly take calculated risks in his attempt to find better solutions to the problems that face mankind. He cannot stick to the safe, the tried and true, the prosaic approaches, and he must pioneer in new areas in a very daring fashion. Creating, unfortunately, also involves destroying. The man who is seeking a new, better solution to an old problem is doing so, in part, because he wants to destroy a present, possibly adequate solution.

Properly motivated and willing to take a chance, the creative worker must, in addition, have self-confidence in his own ability to come up with a new and better solution. This is an extremely important emotional attribute and can only

(Continued on page 27)

* From a paper presented before the ASME Annual Meeting, Chicago, Nov., 1955, and appearing in *Machine Design*, May 3, 1956.

*CONSIDER A CAREER IN ENGINEERING TEACHING**

HAROLD E. WESSMAN

Dean of the College of Engineering

H. E. Wessman

A tremendous amount of publicity in recent months has focused attention on the shortage of engineers and on the competition by industry and by government agencies for young graduates. Every engineering college campus is overrun by trained recruiters coming to interview the seniors and the graduate students. Stories about outstanding students being sought by twenty or more industrial concerns are becoming commonplace. Is it possible that in this intense competition by industry and by government, the opportunities for careers in teaching are being overlooked by our young engineers and scientists?

Engineering colleges do not, and cannot, engage in the highly organized recruiting activities characteristic of large industrial organizations. Nevertheless, enrollments in engineering colleges are increasing rapidly and many new teachers are needed immediately. Many more will be needed in the years ahead. And so, to the young seniors and graduate students with outstanding records, I say, "Do not overlook the fine opportunities open to you in engineering education. Go to your department head or dean and learn how stimulating it is to be an engineering teacher, to work with young men and to help them get the knowledge needed by a professional engineer."

Statistics published by the U. S. Office of Education show that 243,000 students were enrolled in our engineering colleges in the Fall term 1955. Analyses of population statistics and college trends have resulted in a prediction that this engineering enrollment will double by 1967. One might readily conclude that we will need twice as many teachers in 1967 as we have today. However, the increase in graduate study and research programs wherein the ratio of teachers to students is greater than it is for undergaduate programs means that we will need more than twice as many teachers.

A recent survey made by the Relations with Industry Division of the American Society for Engineering Education reports that there are now approximately 8500 engineering teachers in the United States. However, some of these men retire each year because of age, and there are other losses due to health or illness. Consequently, about 10,000 new engineering teachers will be needed in the next ten years. In fact, the ASEE survey just mentioned states that 1300 new teachers are needed for this academic year 1956-57. Obviously, there is no lack of opportunities.

Lists qualities of a good teacher

What are the qualities of a good teacher? Of course, he must know his subject, know it well—so well that he can restate in his own words what others have written into the textbooks he may use in his classes. Moreover, he must know a great deal about supporting areas of knowledge. But do not conclude that a mastery of the knowledge in a particular field must be achieved before one enters teaching. Every teacher knows that he learned more about his subject during his first year of teaching than he did during all of his days as a college student. In fact, if he is a good teacher, he never stops learning. The man in industry, subject to the pressures of an industrial routine, does not usually have the same opportunities to achieve a mastery of knowledge as the man in a university environment.

But there is even more to teaching than knowing one's subject. A good teacher speaks clearly. His explanations are understandable. He shows interest and enthusiasm in his subject and he gets students interested in the subject. He takes an active, personal interest in the progress of his class. He is friendly and sympathetic. A good teacher has good manners, good morals, and a good sense of humor.

What are the elements which make teaching such a stimulating and satisfying experience? Working with young men and watching their minds develop is one of the great sources of satisfaction. In fact, every teacher takes great pride in watching the students who have sat in his classes go out into the world and become successful men and good citizens. Every teacher feels that he has contributed something to that accomplishment.

Teaching is stimulating

Another inviting factor in teaching is the stimulation which comes from a thorough understanding of the limitations of existing knowledge and the acceptance of the challenge to discover new facts through research. There are many gaps and unexplored areas in the accumulation of scientific and technical knowledge. Those individuals in a university environment have unusual opportunities to explore the area of the unknown and to bring light and truth where ignorance prevailed before. It is difficult to describe the joy, the satisfaction, experienced by those who have discovered new principles, or new formulae, or new and better ways to solve problems, or new information about engineering materials and technical processes.

Another element which makes college teaching so attractive is the association with kindred teachers and research workers in many diverse fields. The university is a community of scholars engaged in creative work in teaching, writing, research, or in professional activity. It is difficult to find any other career with the same inspiring environment. Moreover, the opportunity for a fine family social life in a university community is another good reason why many people find happiness and satisfaction in teaching careers.

Salaries on the upswing

How about salaries in engineering teaching? They are not as high generally as salaries in industrial positions. In fact, the salary pattern in many schools is quite unsatisfactory at present. But steps are being taken to remedy this. Many college administrators now recognize that salaries in engineering teaching must be brought to the level of teaching salaries in medicine and law, if the quality of technical education in the United States is to be maintained and improved—in short, if we are to survive in the technological race for world supremacy. Basic salaries in engineering teaching are on the upgrade. Moreover, extra income may

(Continued on page 32)

* Reprinted from *Industrial Science and Engineering*, October, 1956.

April, 1957

The TREND IN ENGINEERING

at the University of Washington

Vol. 9 No. 2

Published quarterly in January, April, July and October, by the Engineering Experiment Station at the University of Washington to reflect the current highlights of its accomplishments and to report progress in its program of investigation and research.

ENGINEERING EXPERIMENT STATION BOARD

Chairman ..... Harold E. Wessman
Dean, College of Engineering
Director of the Engineering Experiment Station ..... F. Burt Farquharson

DEPARTMENT REPRESENTATIVES

Aeronautical Engineering ..... V. M. Ganzer
Chemical Engineering ..... R. W. Moulton
Civil Engineering ..... R. B. Van Horn
Electrical Engineering ..... A. V. Eastman
Forestry ..... B. L. Grondal
Geology ..... H. A. Coombs
Mechanical Engineering ..... B. T. McMinn
Mineral Engineering ..... D. R. Pifer

PUBLICATION COMMITTEE

F. B. Farquharson — R. G. Hennes
B. L. Grondal — W. R. Hill

Editor ..... Isabel W. Hemenway

PUBLICATION OFFICE

127 More Hall
University of Washington
Seattle 5, Washington

Entered as second-class matter March 7, 1949, at the post office at Seattle, Washington, under the Act of August 24, 1912.

CONTENTS

Recognizing and Developing Creative Potential
John E. Arnold, Massachusetts Institute of Technology 1
Reprinted by permission from *Machine Design*, May 3, 1956

Consider a Career in Engineering Teaching
Harold E. Wessman 2
Reprinted from *Industrial Science and Engineering*, October, 1956

A Model Study of Some Fish Facilities for the Proposed Mayfield Dam. Part II: Upstream Migrant System
E. P. Richey, H. H. Chenoweth 5

AEC Grant to Provide Nuclear Equipment . . . 7

The Application of Digital Computers in Power-System Hydro-Storage Problems *David L. Johnson* . . 8

Approximate Laminar Skin Friction in Compressible Flows with Favorable Pressure Gradients
Ely S. Levinsky 13

Noun Phrase Analysis Using the IBM 650 Computer
Aristotelis D. Stathacopoulos 20

Zone Melting: A Physical Method for Controlling Impurities in Metals *Douglas H. Polonis* . . . 23

An Aid in the Preliminary Design of Prestressed Concrete Beams *Jack R. Clanton* 28

Outside U.S.A. 32
An Approximate Solution of Prandtl Equation of Fluid Motion in the Boundary Layer
New Electrolytic Etchants for Stainless Steel Containing Sigma Phase
On the Heat Dissipation of 4-Stroke Cycle Petrol Engines

Left

FIG. 1. Upstream view of model corresponding to sketch shown in Part I.

Below

FIG. 4. Using dye to trace flow distribution.

Right

FIG. 2. Downstream view of model. Fish ladder at left; power-house collection system at right; forebay in upper right.

Above

FIG. 3. Measuring water elevations throughout the model.

A Model Study of Some Fish Facilities for the Proposed Mayfield Dam

Part II: *Upstream Migrant System**

E. P. RICHEY
Associate Professor of Civil Engineering

H. H. CHENOWETH
Assistant Professor of Civil Engineering

E. P. Richey

The basic intent of the upstream system is to attract the migrants from the river into a collection system and provide a ladder for raising them to an elevation above the highest water level where they may be trapped for mechanical conveyance to upstream release points. Reference is made to the January issue of *The Trend* for a sketch of the proposed dam and fish facilities. The barrier across the river will prevent the fish from bypassing the entrances to the collection system. These entrances are provided in the powerhouse between the turbine draft tubes as well as near the junction of the powerhouse and the barrier. The ladder begins at the end of the powerhouse and makes a 180° bend, finally to terminate in the traps at the base of the "upstream facility."

Figure 1 shows an over-all photograph of the model, built to a scale of 1:10, and was taken from the same angle as shown in the sketch; in the prototype, the section in the foreground would be contained within the powerhouse. The entrances to the fish-collection channel are shown discharging into the tailrace. The main entrance and the barrier appear in the upper right corner, and the ladder portion occupies the background. A view of the model with the opposite perspective is shown in Fig. 2. The ladder ends near the manometer board in the center of the picture. (See also Fig. 3.)

In the hydraulic sense, the upstream facility is a complicated open channel that is required to maintain certain standards of flow throughout the operating range of tailwater elevations. The amount of water delivered must be varied with tailwater in order to keep the velocities in the transportation system reasonably constant. For instance, at high tailwater, the correct quantities would have to be introduced through diffusion gratings into the transportation section at seventeen different points. The water for operating the upstream system is to be pumped from the river rather than drawn from the reservoir behind the dam. Thus, the amount of water necessary and the elevation it must be lifted became important items in the design. The purpose of the model was to check the initial design layout and to observe flow conditions for different arrangements of the basic components of the system.

H. H. Chenoweth

The specified design flow rates must be discharged from the several entrances in order to attract the fish into the collection channel. When the fish are once in the channel, the depth and velocity of the water must be conducive to their upstream movement. In order to maintain the required velocities in the powerhouse collection channel with a minimum amount of water, various types of baffles were used in the powerhouse section. The model indicated the flow pattern (Fig. 4), making a comparison of the different baffles relatively easy. One of the series of baffle arrangements is shown in Fig. 5.

The powerhouse collection channel and the main entrance bring the fish to the ladder. Each pool in the ladder series shown in Fig. 6 provides a one-foot rise in elevation. The fish may pass over the notched weir or through a port in the weir plate, not visible in the photograph. In order to provide additional water to maintain the proper velocity with increasing tailwater elevation, a supply channel is provided between the two legs of the U-shaped ladder. The added water flows from the supply channel over the vertical plates, or "chimneys," on the side of the

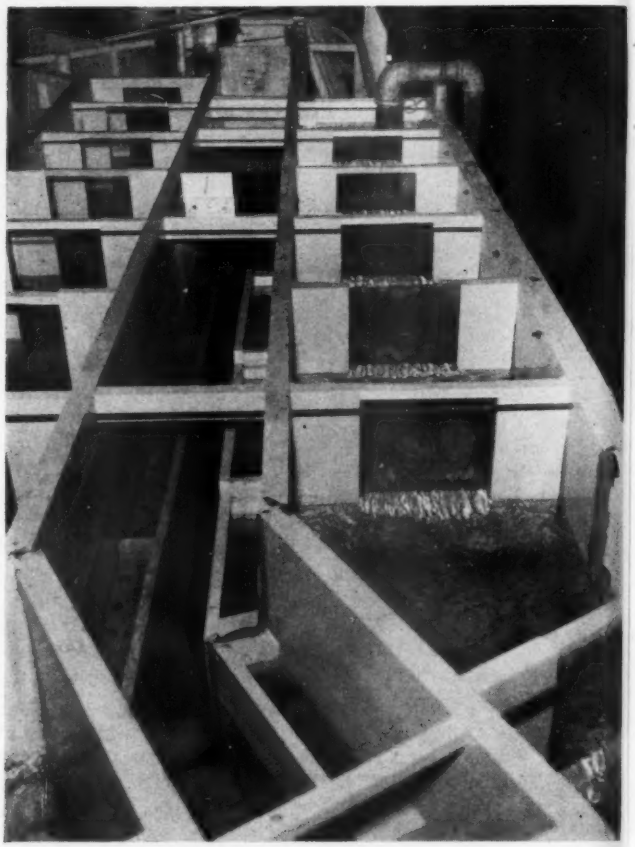
* The Upstream Migrant System, another part of the fish facilities for the proposed Mayfield Dam on the Cowlitz River, was studied in model scale at the Charles W. Harris Hydraulics Laboratory. This article is a sequel to Part I: "The Skimmer" that appeared in the January issue of *The Trend*. Both studies were carried out on models designed and built by the Major Projects Division of the Tacoma Department of Public Utilities.

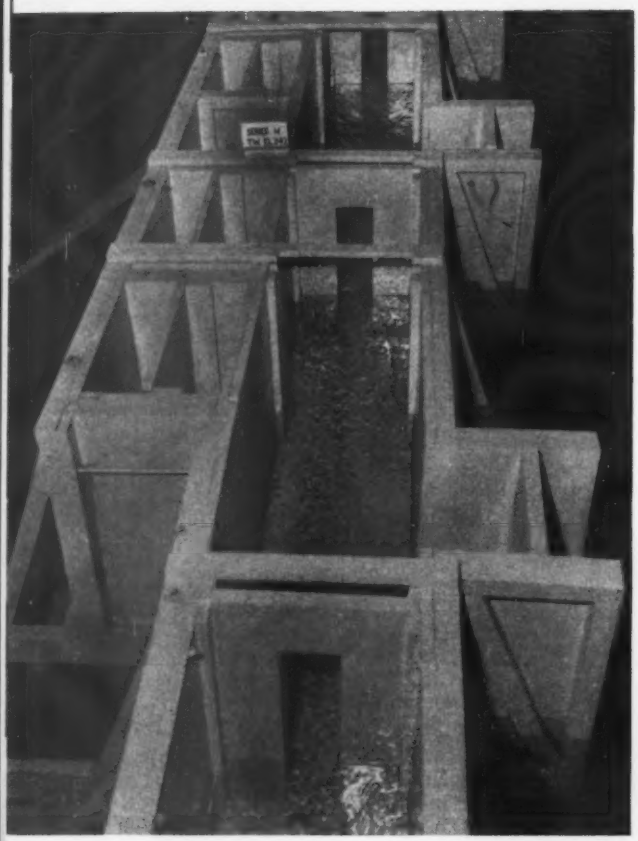
Below

FIG. 6. One of the ladder series showing orifices.

Left

FIG. 7. Initial design of the ladder bend.





Above

FIG. 5. View of one type of baffle.

Right

FIG. 8. Final design of the ladder bend.

channel, then downward and through a grating to discharge laterally into the ladder proper. In the initial design, the flow through the chimneys as well as through other parts of the system was not uniform. A basic revision was made by inserting flow-metering orifices between the supply channels and the collection and transportation sections. The orifices were dimensioned to increase the head differential needed to deliver the desired flow rates from a few hundredths of a foot to one foot. The resulting rates were thus made much less sensitive to minor fluctuations in surface elevations. The orifices between the supply channel and the ladder also are shown in Fig. 6. Similar orifices were located upstream from all other diffusion gratings. The orifices can be placed at different elevations to function quite automatically to meet the variable flow demands as the tailwater elevation changes. Their stabilizing effect on the delivery of water to the various parts of the facility, however, comes at the sacrifice of increasing the head against which the pumps must operate.

The model did not disclose any new hydraulic concepts, nor was this its intent. It did provide a rapid way of showing how a wide variety of different control features could be brought together which would be impractical to attempt by theory alone. A fish facility presents very important aspects of flow conditions that are of little consequence to the usual hydraulic engineering problems. One example of this is brought out in the two photographs of the bend in the ladder. In the initial design (Fig. 7) an upwelling occurred on the curved wall. So far as the pure hydraulics is concerned, this feature is not significant. However, such a current adversely attracts fish to delay their upstream migration, so it must be corrected. A reorientation of the transverse weir plate to the position shown in Fig. 8 minimized the undesirable upwelling.

Another feature, undesirable from any viewpoint, would be the development of a surface wave train extending through the notched weir plates. Attempts to incite such a wave pattern in the ladder section were unsuccessful, leading to the conclusion that the types of weirs installed and the relative dimensions of the ladder would inhibit the formation of such a wave pattern. Thus, the model furnished a means of meeting the need for visual and qualitative information on the over-all performance as well as the usual quantitative measurements to compare the performance with the design standards.

AEC Grant to Provide Nuclear Equipment

THE COLLEGE OF ENGINEERING has been granted $109,500 by the Atomic Energy Commission for the purchase of nuclear laboratory equipment and materials. The grant will be used to equip the Reactor Theory, Chemical Engineering, Nuclear Metallurgy, and Radioactive Waste Disposal laboratories.

The major unit in the Reactor Theory Laboratory will be a subcritical graphite exponential assembly similar to the one at the Argonne National Laboratory. Parts for a reactor simulator, which will be built at the University of Washington, will also be obtained with the grant.

The major part of the grant will be used for a number of small items, such as scalers, counters, and survey meters with which all the laboratories must be equipped. The Chemical Engineering Laboratory will also be equipped with a spectrophotometer, a scintillation spectrometer, a sample changer, and filter hoods.

Equipment for the Nuclear Metallurgy Laboratory will be expanded to include a metallograph, a dilatometer, a vacuum furnace, a heat-treat furnace, and an abrasive cutoff wheel.

A graduate program in Nuclear Engineering leading to a Master of Science degree was initiated in the fall of 1956. It is a cooperative undertaking of the Departments of Chemical, Civil, Electrical, Mechanical, and Metallurgical Engineering and is administered by a faculty committee composed of representatives from these departments under the chairmanship of Dr. R. W. Moulton, Executive Officer, Department of Chemical Engineering. The following courses are now offered.

Introduction to Nuclear Engineering
Nuclear Instruments
Nuclear Reactor Theory
Nuclear Reactor Theory (Advanced)
Nuclear Engineering
Heat Transfer
Nuclear Power Plants
Tracer Techniques in Mechanical Engineering Measurements
Nuclear Metallurgy
Nuclear Metallurgy Laboratory
Control of Radioactive Wastes
Radiochemistry
Radiochemistry Laboratory

THE APPLICATION OF DIGITAL COMPUTERS IN POWER-SYSTEM HYDRO-STORAGE PROBLEMS

DAVID L. JOHNSON

Assistant Professor of Electrical Engineering

D. L. Johnson

Determination of the most economical use of hydro storage, either for an individual power system or in power-pool operation, has long been a problem solved by an impressive array of trial-and-error solutions. Each trial solution has required extensive hand calculation and use of numerous lengthy tables. A considerable practical background involving familiarity with the individual system is necessary in order that the number of separate solutions not become prohibitive. The usual method of solution of hydro-storage problems has been to start with an "educated guess" or engineering assumption; then by lengthy trial corrections, a solution is obtained that is hoped to be the optimum one under the prescribed conditions.

Generally, this type of solution leaves much to be desired. It imposes considerable responsibility upon the engineer who is establishing the starting values, it requires extensive and repetitive calculation, and often gives little assurance that the final answer will be the most satisfactory one.

In the spring of 1955, a cooperative study was undertaken by Seattle City Light and the University of Washington Department of Electrical Engineering, one phase of which was concerned with the investigation of the Seattle City Light hydro-usage problems by digital computer means. The computer facilities of both organizations at that time were IBM 604 electronic calculating devices.

The basic problem investigated has been a limited one; however, the methods and techniques developed can well be applied to more complex systems.

Problem Definition

The particular problem involved is one concerning the hydro-storage of the combined Seattle and Tacoma systems in operation as a part of the Northwest Power Pool under a Bonneville Power Administration (BPA) contract. Each utility is required to submit annually a "computed demand table" which considers estimated peak and average loads and generating capabilities to determine monthly peak and average power requirements from BPA for the forthcoming year. The calculations to fix the generating capabilities must assume minimum water-year conditions. The problem thus becomes one of accurate estimated loads and economical hydro usage. This discussion will not be concerned with load estimates, although this is an area in which digital computers could easily be used to advantage.

The contract is so set up that a utility may not be credited for covering a pessimistic load estimate from its own facilities even though it may have the hydro capability to do so. The contract cost of BPA power to each utility will be dependent upon the maximum power demand of that utility, either peak or average, as shown by the computed demand table, which then becomes a theoretical device based upon actual estimated loads and a standard minimum water year. The accuracy of such a study and the requirement that the system use the minimum water-year capabilities to the optimum is a problem of extreme economic significance during a given contract year. Penalty clauses in the contract consider the possibility of underestimating the maximum power required of BPA by the utility.

At present in the combined Seattle-Tacoma systems, there is only one major storage reservoir which is used economically for variable-head storage. The other reservoirs are primarily suited for maintaining nearly a fixed head for maximum peaking capability. The variable Ross reservoir, however, feeds not only into the facilities at Ross plant, but downstream on the Skagit River to two other generating facilities,

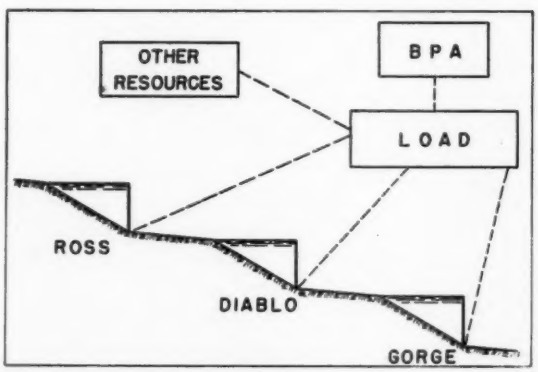


FIG. 1. SCHEMATIC REPRESENTATION OF THE SKAGIT HYDRO SYSTEM

Diablo and Gorge plants. A schematic representation of the Skagit hydro system is shown in Fig. 1.

Over a monthly period the heads at the downstream reservoirs remain essentially constant. The problem breaks down into one of a single variable: storage reservoir. It should be noted that the generating facilities on the Skagit River supply roughly one-half of the total combined system generation.

As the cost per megawatt-hour of BPA power is dependent upon the use factor and therefore is a function of the maximum power deficit of the utilities as shown by the computed demand table, it is most economical to make the maximum peak power deficit equal to the maximum average energy deficit during the month in which this deficit occurs. Keeping the monthly average energy deficit equal to the maximum value during the withdrawal period in the months preceding the month of the maximum peak deficit will maintain maximum storage and therefore permit the lowest possible value of maximum peak and average power deficit during the contract year.

Means of Solution

The methods of solution of the problem previously used by Seattle City Light engineers was one set by the BPA. The general criteria previously mentioned were followed in a repetitive trial-and-error solution. In light of experience, certain withdrawals were used in the early months of contract calculation. (The contract period extends from July through the following July.) An estimated value of the maximum deficit and the month in which this deficit would occur was assumed. Calculation was then made to determine the storage required from Ross reservoir to meet this maximum value. When the calculations were complete to the assumed month of the maximum deficit, and the average energy deficit and peak power deficit were neither equal to each other nor equal to the assumed value, a correction was made upon the assumed value and the sequence was recalculated. Each of the numerous calculations required the use of extensive tables and six-place accuracy. After the correct results were finally obtained to the month of the maximum, which often falls in January, the process was generally repeated for successive months under certain limits until the final result was complete refill of Ross reservoir by the end of the contract period. Such a method required numerous man-hours and was seldom free from the consideration that a numerical error may have invalidated the whole calculation.

When this problem was approached from the standpoint of a digital computer solution, two general courses were possible. One was simply to mechanize the required calculation by use of the computer; the other was to investigate the basic theory of the over-all problem and try to improve the method as well as the means of solution. The latter course was followed, and a simple method leading to a more exact solution was obtained.

The value of the average power output of the Skagit system can be expressed as a function of Ross reservoir storage. The constants involved are the initial storage at the beginning of the period of calculation, the minimum water-year inflows of the Skagit averaged over the period of calculation, and the information relating the quantity of water released at a given head to the quantity of power produced. This last information had previously been taken from extensive tables relating head to H/K (megawatts per cubic foot-second). For use on the computer the method of least squares was used to obtain H/K as a function of reservoir storage. The results obtained by this method were very satisfactory —probably as accurate as the original data. This operation was required only at Ross plant, since the head and H/K were assumed to be fixed at the downstream plants.

The power output of the Skagit system as averaged over d days can be expressed as

$$\begin{aligned} P_{Sk} &= P_R + P_D + P_G{}^* \\ &= F_R\,(H/K)_R + F_D\,(H/K)_D + F_G\,(H/K)_G \\ &= \left(NF_R + \frac{Q_0 - Q_e}{d}\right) f\left(\frac{Q_0 + Q_e}{2}\right) + \left(\frac{Q_0 - Q_e}{d}\right) \times \\ &\quad [(H/K)_G + (H/K)_D] + M \qquad (1) \\ &= aQ_e^2 + bQ_e + cQ_0^2 + dQ_0 + e\,, \end{aligned}$$

where F is the total plant flow and NF is the natural flow under minimum water-year conditions, Q_0 and Q_e are Ross reservoir storage in second-foot-days, at the beginning and end of the period of calculation (d days), respectively, and H/K is the average H/K over the period, represented as a linear function of average reservoir storage.

Assuming that the Skagit power is the only system variable, and that the capabilities of the rest of the system are fixed and known during each period, the average energy deficit for a given period can be determined:

$$\begin{aligned} P_{\text{av def}} &= L_{\text{av}} - P_{\text{av Sk}} - P_{\text{av fix}} \\ &= -aQ_e^2 - bQ_e - cQ_0^2 - dQ_0 + E\,, \qquad (2) \end{aligned}$$

where L_{av} is the total average predicted load for the system over the time period, and P_{av} is the total average energy capability of the system over the time period broken down into the variable, $P_{\text{av Sk}}$,

* Subscripts Sk, R, D, and G refer to Skagit, Ross, Diablo, and Gorge, respectively.

and the fixed power capability of the system other than the Skagit power, $P_{av\ fix}$.

The peaking capability of the system is assumed to be fixed and known for all of the facilities other than Ross plant. This is a valid assumption for normal operating conditions. The information regarding peaking capability has previously been used in tabulated form relating peaking capability to head. As in the case of the Ross H/K factor, the method of least squares was used to determine the peaking capability of Ross plant as a function of reservoir storage. This was done in the form of a quadratic,

$$P_{peak\ R}=AQ_{min}^2+BQ_{min}+C\ , \qquad (3)$$

where Q_{min} is the minimum storage in the Ross reservoir during the time period.

The peak power deficit can now be expressed in the form:

$$\begin{aligned} P_{peak\ def} &= L_{peak}-P_{peak} \\ &= L_{peak}-P_{peak\ fix}-P_{peak\ R} \\ &= -AQ_{min}^2-BQ_{min}+D\ , \end{aligned} \qquad (4)$$

where L_{peak} is the total peak predicted load for the system over the time period, P_{peak} is the total system peak capability over the time period, made up of the variable $P_{peak\ R}$ and the fixed peaking capability of the system other than Ross, $P_{peak\ fix}$.

Equations 2 and 4 are oversimplified in that they do not consider the average and peak power reserves allowed under contract agreement. In the case of the average power, the reserve is a function of the steam capability of the system and therefore can be incorporated into $P_{av\ fix}$. The peak power reserves allowed are more complex in that they may be fixed by either a percentage of the total peaking capability or by the loss of the largest machine, whichever is the lesser. This problem involves double solutions of equations, in many instances using two sets of coefficients and a decision as to which should be accepted, but does not alter the general structure of the solution. In summary, it can be stated that both the peak power and average energy deficits can be expressed in terms of the Ross reservoir storage, average predicted loads, average and peak generating capability of the combined systems other than the Skagit, and the average stream flows as fixed by the minimum water-year. This is done by means of quadratic equations which have fixed coefficients for any given period of time and given system. The value of Ross reservoir storage is the only variable.

One of the criteria previously mentioned was that of maintaining the average energy deficit equal to the value of the maximum peak power deficit from the time of withdrawal to the period when the maximum peak deficit is reached. This can be mathematically expressed by operating upon Eqs. 2 and 4 in the following way. The monthly peak power deficit, Eq. 4, for the final month of the period under calculation is equated to the mean of the average energy deficits over the period starting with the beginning of the drawdown and terminating upon the final month. This means equating Eqs. 4 and 2, using coefficients of the final month in (4) and coefficients averaged over the period of calculation in (2). This operation will result in a single quadratic equation for each period starting with the month in which withdrawal is started and ending upon successive months until a maximum deficit value is determined. The result of this operation will be the minimum peak power deficit for the annual period and the value of the average energy deficit that is required from withdrawal to the month of the maximum peak deficit to hold this value at its minimum.

When the average energy deficit to be used over this period has been obtained, monthly calculations are made, using Eq. 2 to determine the Ross storage required at the end of each month through the month of the maximum peak deficit, and Eq. 4 to give the resultant peak power deficit for each month. A simple routine to determine whether Q_0 or Q_e is equal to Q_{min} is used before Eq. 4 can be run.

During the periods immediately following the month of peak deficit, the loads are still high and the inflows are low, so the peak and average deficits are again equated on a monthly instead of an average basis, and the storage required to give these deficit values is calculated by using a quadratic equation made up by combining Eqs. 2 and 4. This storage is then used in the peaking Eq. 4 for determination of the value of the peak and average deficits.

After this period, the primary consideration is that of reaching total refill of the Ross reservoir before the beginning of the next annual withdrawal period. The determination of which method should be used to accomplish the refill is subject to many limitations. The reservoir cannot be withdrawn beyond a certain critical level, the deficits cannot at any time exceed the maximum, and a certain minimal flow must be maintained at all times. These criteria require many decisions and the possibility of several recalculations. For instance, if the critical level of the reservoir is exceeded or the minimal stream flow is not maintained under one method of withdrawal or refill following the month of the maximum, another modified method must be used. These choices can be, and of course for computer application must be, systematized in such a way that one negating result suggests another means of solution to the computer.

A sample tabulation of the per-unit results obtained by digital solution of the computed demand problem is shown in Table I.

TABLE I

PER-UNIT RESULTS OBTAINED BY DIGITAL SOLUTION OF THE COMPUTED DEMAND PROBLEM

Month	Month-End Ross Reservoir Storage	Skagit Capability Power		Combined System Deficit	
		Peak	Average	Peak	Average
July	1.000	2.300	1.009	-0.555	0.318
August	1.000	2.300	0.633	-0.374	0.905
September	0.953	2.271	0.709	-0.141	1.000
October	0.888	2.224	0.751	0.287	1.000
November	0.780	2.141	0.921	0.681	1.000
December	0.695	2.078	0.877	0.791	1.000
January	0.556	1.956	1.059	1.000	1.000
February	0.421	1.818	1.011	0.988	0.988
March	0.361	1.744	0.732	0.889	0.899
April	0.310	1.653	0.972	0.645	0.437
May	0.493	1.653	0.921	0.385	0.437
June	0.924	1.897	0.858	0.173	0.437
July	1.000	2.244	1.161	-0.267	0.437

Computer Application

The method of solution outlined above was initially programmed for the type 604 IBM computer. The complete solution of the sequence over the contract year was accomplished by the solution of from 15 to 20 quadratic equations and 13 to 15 equations involving only simple substitution. It should be recognized that coefficients must be precalculated for each of the monthly peaking Eq. 4's; for the monthly quadratics from which storage is calculated, Eq. 2; and for the quadratics which result from equating Eqs. 2 and 4. These coefficients, however, once calculated, will remain constant as long as the storage facilities of the system are unchanged.

Calculation of the problem on the IBM 604 was accomplished in two sections, using a single set of computer panels. The two sections are specified as the "data-processing" section and the "operation" section. The data-processing section is a single run in which the system data are put into a form suitable for solution of the various equations and punched into the operation cards, which have previously been partially punched with precalculated fixed coefficients. In order to have consistent monthly programs, an operation card is prepared for each possible type of solution for every month. This means that, in most months, only two or three out of six operation cards will actually be used in the operation section. However, it is not known before the operation run is started which of the various operation cards will be used for a given month. It should be emphasized that the only variable information to be punched on the data-processing cards will be that of estimated loads and outside system capacity. None of this need be precalculated.

Following the data-processing run, an operation run is made to determine the month and value of the maximum deficit. The maximum deficit value is then gang-punched into the monthly operation cards through the month of the maximum, and a run is made to fix the monthly storage used and the peak power deficit through this period. If the month of the maximum deficit falls in January, the February and March run is made by equating the average energy deficit and the peak power deficit in order to determine this value and the storage used for these months. A calculation is next made to determine the constant average energy deficit required to give refill during the April-to-July period. This calculation is in the form of Eq. 4. By using this value of average deficit, the monthly values of storage used (negative in this case) and peak power are determined for each month.

Unusual load schedules can create several variances in this solution. If the method summarized should require more storage withdrawal than is available during April, a new average deficit must be calculated for that month in order to stay within system limitations. This equation again has the form of Eq. 4. This value might possibly be greater than the maximum deficit value, in which case the solution must be modified from February through April. This may be done by finding a constant average energy deficit for this period to result in complete withdrawal by the end of April. It is also possible that unusual load conditions might occur which would result in the February peak and average deficits being relatively low, with the March peak and average deficits higher than the maximum. In instances such as this, a constant average energy deficit which is below the maximum is determined and held through these two months. Provision is made for such possibilities in the program, and operation cards are punched and processed in the initial data-processing run to be prepared for these eventualities.

The total solution of the problem for the contract year using the IBM 604 requires slightly less than one hour. The operating instructions are such that no familiarity with the problem is required by the operator. The average error for the year tested was about 0.05 per cent, with a maximum error of less than 1.50 per cent. When a number of cases of the type discussed had been run, it became increasingly evident that, while the 604 offered a practical solution to the problem, the use of this machine required extensive card handling, value comparisons and evaluations by the operator, and several operator decisions. These decisions were not those of a technical nature, but of the order of "If A is bigger than B, do this; if B is bigger than A, do this; if A equals B, do this." The 604 is certainly not the most

efficient machine by which to obtain a solution to a problem of this type. Therefore a solution using the larger and faster IBM 650 was considered.

The IBM 650 with its 2000 words of storage and sizable order code is of such a magnitude that the hydro-storage problem can be solved from start to finish, using a stored program with no operator intervention. Because of the fact that the computer must make its own decisions and comparisons and chart its own course through the entire problem solution, numerous variations in the application of the general solution method are required. These variations, however, do not affect the general solution method as it has been discussed, but only the means of presenting this method to the computer.

The computer program is so set up that the system variables are supplied to the machine as a part of the input, and the machine is started through the program. The device itself will determine the course of calculations to be taken from each preceding operation, evaluate the results, and stop only when an acceptable annual computed demand table has been obtained and punched out from the computer. At the time of this writing, the entire program has been organized and the section comparable to the operation run on the 604 has been smoothed out and is in operation. From the results of this section of the over-all program it can be predicted that the total elapsed time required for a complete solution of the annual hydro-storage problem of the system studied will be approximately ten minutes. The accuracy will be slightly better than that available by means of the IBM 604. The limitation upon the accuracy is not one of possible significance in calculation, but that imposed by the mathematical representation of the tabulated material. If there were any necessity for higher ranges, the functional mathematical representations could be carried to greater accuracy or the table-lookup potential of the 650 could be used. These possibilities will be investigated if it appears that an improvement in accuracy of over-all results would be advantageous. Since much of the input information is only significant to three places, it seems doubtful that there is any merit in obtaining results to more than five places of accuracy.

It must be recognized that the particular problem under study is a limited one. The computed demand table does not specify actual system drawdown procedure, but is made under contractual specifications for the express purpose of satisfying certain contract requirements. The method used, however, could easily be extended to include actual operating condiditions and a greater number of variable and related reservoirs. Familiar computer methods for the solution of simultaneous equations might well be incorporated into the more complex solution. Addition of certain specifications and limitations could also make the program adaptable for actual drawdown solution. In instances where tailwater elevation is a significant factor, this problem could be incorporated into Eqs. 2 and 4 without changing the basic structure of the solution.

As the problem is extended in scope, the solution will become considerably more complicated. The IBM 650 solution time is divided between actual calculation and decision operations; of these two sections, the decisions and the attendant changes in program take up the greater time. With the extended problem, the calculations will increase, for the solution of simultaneous equations will be required; but the number of decisions will rise very sharply with the resultant increase in time requirements. It can be estimated that the time required by a computer solution of a hydro-storage problem may rise with the number of variable reservoirs between a linear and square relationship.

The means of rapid calculation of withdrawal problems gives rise to a number of studies concerned with things other than contract calculation. Mass studies of the effect of weather conditions might be of considerable service in evaluation of the effect of alternate load estimates such as those used in the contract calculations. Many changes in system require only minor changes in the precalculated coefficients, and the proposed method could be an integral part in analysis of future planning. Extensions of the method could be used to obtain exact solutions for control of large hydro networks or combinations thereof.

Conclusion

The solution of the problem involving economical use of hydro storage is one that lends itself well to digital computer solution. The preparation of any solution for computer application should be more than the mere mechanization of methods previously used, but must require a re-examination of the basic problem in terms of the most elementary mathematics and theory. This fact is well borne out in the ease of solution as demonstrated in this particular problem. The final method was certainly not the one with which the investigator initially approached the problem. As familiarity with the problem developed, however, simplifications became apparent, and the final method of exact solution gradually evolved. Because of the nature of the problem and the tremendous number of solution limits involved, the application of the general method of solution to a specific computer solution requires a frustratingly great number of computer decisions and variations. This can be handled moderately well, however, on a medium-sized digital computer such as the IBM 650

(Continued on page 32)

Approximate Laminar Skin Friction in Compressible Flows with Favorable Pressure Gradients*

ELY S. LEVINSKY

Graduate Student in Aeronautical Engineering†

E. S. Levinsky

Introduction

Laminar skin friction with zero pressure gradients has been the subject of many papers. The classical solution was obtained by Pohlhausen[1] for incompressible flow by representing the velocity profile in the boundary layer by a power series in y/δ, in which a parameter is used to account for the effects of pressure gradient. The accuracy of Pohlhausen's solutions was shown by Thwaites[2] to be surprisingly good for favorable pressure gradients; however, for adverse pressure gradients, where the possibility of separation exists, the agreement was poor. Thwaites modified Pohlhausen's procedure, improving its accuracy for the latter case. However, even this improved method failed to accurately predict separation. The present procedure is based on Thwaites' method, and therefore is valid only for favorable pressure gradients.

The calculation of compressible skin friction with pressure gradient was greatly simplified when, in 1949, Illingworth[3] and Stewartson[4] introduced a transformation of coordinates which reduced the compressible flow equations to the incompressible form, thus enabling incompressible methods to be used in obtaining a solution. Rott and Crabtree[5] in 1952 used this transformation, along with the method of Thwaites, to calculate compressible skin friction.

In 1953, Rott[6] generalized the Illingworth-Stewartson transformation, making it valid for any constant Prandtl number near unity. Following a procedure similar to that of Rott and Crabtree, this generalized transformation is used in the present method of calculating skin friction. The viscosity law, $\mu_0/\mu = C(T_0/T)$, first used by Chapman and Rubesin,[7] is employed, rather than the more approximate relation, $\mu_0/\mu = T_0/T$, used by Rott and Crabtree.

Many of the derivations presented in the discussion are in a highly abbreviated form.[8]

Nomenclature

a = velocity of sound

a' = velocity profile parameter $= \dfrac{\partial U/U_1}{\partial Y/\Delta}$

B_1 = parameter defined in Eq. 8

c = airfoil chord length

C = constant in the linear viscosity law defined by Eq. 7b

c_{f_1} = local skin friction coefficient based on local free stream dynamic pressure $= \tau/\frac{1}{2}\rho_1 u_1^2$

c_{f_∞} = local skin friction coefficient based on undisturbed free stream dynamic pressure $= \tau/\frac{1}{2}\rho_\infty u_\infty^2$

C_{F_∞} = average skin friction coefficient based on undisturbed free stream dynamic pressure, defined by Eq. 27

C_P = specific heat at constant pressure

G = function of a', defined by Eq. 17

H = ratio of transformed displacement to momentum thickness, defined by Eq. 9

J = function of a', defined by Eq. 19

k = thermal conductivity

K = function of a', defined by Eq. 9

l = function of m, defined by Eq. 12

L = function of m, defined by Eq. 14

m = boundary layer parameter, defined by Eq. 13

M = Mach number $= u/a$

p = pressure

Q = function of a', defined by Eq. 18

R = gas constant

Re_1 = Reynolds number based on local free stream conditions and distance x

$Re_{\infty c}$ = Reynolds number based on undisturbed free stream conditions and length c

T = temperature

u = velocity component parallel to surface

v = velocity component normal to surface

U, V = velocity components in transformed flow, defined by Eq. 5b

*Arranged by the author from his M.S. thesis of the same title and his paper which won the regional award given by the Institute of the Aeronautical Sciences, West Coast Conference, Graduate Division, 1956, published by the IAS.

†The author was employed by Boeing Airplane Company at the time he obtained his degree of M.S. in Aeronautical Engineering, 1956. He is now a senior aerodynamics engineer with the San Diego Division of Convair.

x = curvilinear coordinate along surface
y = coordinate normal to surface
X, Y = coordinates in transformed flow, defined by Eq. 5a
γ = ratio of specific heats
δ = boundary layer thickness
Δ = boundary layer thickness in transformed flow
Δ^* = displacement thickness in transformed flow, defined by Eq. 11
η = small positive constant
θ = momentum thickness in physical coordinates
Θ = momentum thickness in transformed flow, defined by Eq. 10
Θ^* = function of U/U_1, defined by Eq. 20
λ = Pohlhausen boundary layer parameter $= (\delta^2/\nu)\ (du_1/dx)$
μ = absolute viscosity
ν = kinematic viscosity $= \mu/\rho$
ξ = function of Prandtl number $= 1-\sqrt{\sigma}$
ρ = density
σ = Prandtl number $= \mu C_P/k$
τ = shear force per unit area
Φ = function of velocity
Ω = momentum thickness parameter, defined by Eq. 22

Subscripts

1 = condition at outer edge of boundary layer
∞ = undisturbed free stream conditions
w = conditions on surface
0 = reference condition in free stream
c = reference length based on chord
FP = flat plate

Discussion

We shall begin with the Prandtl boundary layer equations for two-dimensional laminar flow. They are the equation of continuity, the momentum equations in the x and y directions, the energy equation, and the equation of state, which are listed in this order as

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0\,, \tag{1a}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=u_1\rho_1\frac{\partial u_1}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{1b}$$

$$\frac{\partial p}{\partial y}=0\,, \tag{1c}$$

$$\rho u C_P\frac{\partial T}{\partial x}+\rho v C_P\frac{\partial T}{\partial y}=-uu_1\rho_1\frac{\partial u_1}{\partial x}+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2, \tag{1d}$$

$$p=\rho RT\,. \tag{1e}$$

The energy equation (1d) may be considerably simplified by taking the Prandtl number $\sigma=\mu C_P/k=1$. By multiplying the momentum equation (1b) by u and adding the result to (1d), and then making the additional restriction of zero heat transfer, the energy equation reduces to the same form as in non-viscous flow,

$$\frac{T}{T_1}=1+\frac{\gamma-1}{2}M_1^2\,(1-u^2/u_1^2)\,. \tag{2}$$

Tani[9] assumed that a relation somewhat similar to Eq. 2 might be used to replace the energy equation, even when the Prandtl number was not unity. He used the expression,

$$\frac{T}{T_1}=1+\frac{\gamma-1}{2}M_1^2\left[\sqrt{\sigma}\left(1-\frac{u^2}{u_1^2}\right)+\Phi\xi\right], \tag{3}$$

where $\xi=1-\sqrt{\sigma}$, and Φ is a function of u/u_1 only, which must have the values, $\Phi(0)=\Phi(1)=0$, if the temperature is to satisfy the boundary conditions. Equation 3 will be used in the present analysis, restricting the results to flows with zero heat transfer, but with arbitrary pressure gradients.

The function Φ was obtained by Tani, who represented it by a power series in u/u_1 and then solved for the coefficients. He also stated the following integral relation which Φ must satisfy,

$$\int_0^\delta \rho u\Phi dy=\int_0^\delta \rho u(1-u^2/u_1^2)dy\,. \tag{4}$$

Modified Illingworth-Stewartson Transformation

For incompressible flow, the boundary layer equations may be solved by integrating the momentum equation across the boundary layer. Such an integral procedure would be difficult to apply to the compressible case with the boundary layer equations in their present form, because of the appearance of ρ, T, and μ, in addition to u and p, as the basic flow variables.

Illingworth[3] and Stewartson[4] introduced a transformation which reduced the continuity and momentum equations to the same form as for incompressible flow. Their transformation was limited to a Prandtl number of one. Rott's[6] modified Illingworth-Stewartson transformation, which is valid for any Prandtl number, will be used in this paper, increasing the generality of the results. His transformation may be written

$$\frac{\partial}{\partial x}=\left(\frac{T_1}{T_0}\right)^{\frac{\gamma}{\gamma-1}+\frac{\sqrt{\sigma}}{2}}\frac{\partial}{\partial X};\frac{\partial}{\partial y}=\left(\frac{T_1}{T_0}\right)^{\frac{\sqrt{\sigma}}{2}}\frac{\rho}{\rho_0}\frac{\partial}{\partial Y}, \tag{5a}$$

and

$$U=\left(\frac{T_0}{T_1}\right)^{\frac{\sqrt{\sigma}}{2}}u\ ;\ V=\left(\frac{T_0}{T_1}\right)^{\frac{\gamma}{\gamma-1}+\frac{\sqrt{\sigma}}{2}}\frac{\rho}{\rho_0}v\,, \tag{5b}$$

where x and y are the coordinates in the original flow, and X and Y are the corresponding coordinates in the transformed flow. The subscript 0 refers to a reference point in the free stream, and therefore the isentropic relation, $\rho_1/\rho_0=(T_1/T_0)^{\frac{1}{\gamma-1}}$, holds outside the boundary layer.

The continuity equation (1a) may be expressed in a form similar to that for incompressible flow by applying the transformation equations, (5a) and (5b). Thus, in the transformed coordinates it becomes

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0\,. \tag{6}$$

The momentum equation in the x direction (1b) may also be transformed in a similar manner. First, however, it is necessary to introduce the viscosity-temperature law,

$$\frac{\mu_0}{\mu}=C\,\frac{T_0}{T}\,, \tag{7a}$$

which was used by Chapman and Rubesin[7] in their work on flat plates, where the variable C is defined by the highly accurate Sutherland law,

$$C=\left(\frac{T_0}{T}\right)^{\frac{1}{2}}\frac{T+216}{T_0+216}\,. \tag{7b}$$

Now, by utilizing Eqs. 5a, 5b, and 7a, the momentum equation becomes, after some manipulation,

$$U\frac{\partial U}{\partial X}-\frac{\left(\frac{T}{T_1}\right)+\left(\frac{\gamma-1}{2}\right)\sqrt{\sigma}\,\frac{u^2}{a_1^2}}{1+\left(\frac{\gamma-1}{2}\right)\sqrt{\sigma}\,M_1^2}\,U_1\frac{dU_1}{dX}$$

$$+V\frac{\partial U}{\partial Y}=\nu_0\frac{\partial\left(\frac{1}{C}\frac{\partial U}{\partial Y}\right)}{\partial X}\,.$$

Substituting for T/T_1 from the revised energy equation (3), this reduces to

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\left[1+B_1\xi\Phi\right]U_1\frac{dU_1}{dX}+\nu_0\frac{\partial\left(\frac{1}{C}\frac{\partial U}{\partial Y}\right)}{\partial Y}\,, \tag{8}$$

$$\text{where } B_1=(\gamma-1)\,(M_1^2/2)\left[1+\sqrt{\sigma}\,\frac{\gamma-1}{2}M_1^2\right]^{-1}.$$

Equation 8 is the momentum equation in the transformed flow. It is of the same form as given by Tani.[9] The variable temperature, density, and viscosity which appeared in the original form (1b) have been eliminated. Except for the appearance of Φ and the deviation of C from unity, Eq. 8 is identical with the incompressible momentum equation.

Integration of the Momentum Equation

The solution of Eqs. 6 and 8 can be accomplished by formally integrating Eq. 8 across the boundary layer, the relationship expressed by the continuity equation (6) being used in the process. The resulting integrals are then evaluated by representing the velocity U by a power series in Y/Δ, where Δ is the boundary layer thickness in the transformed flow. Because of the appearance of Φ and C in the momentum equation, formal integrals involving these functions will also occur. Tani evaluated these integrals by representing Φ and C also by power series in Y/Δ, and some of his results will be employed in the following analysis.

By integrating Eq. 8 from $Y=0$ to $Y=\Delta$, setting the reference temperature $T_0=T_w$, and making use of the continuity equation (6), the momentum equation becomes

$$\frac{U_1}{\nu_0}\frac{d\Theta^2}{dX}+2\left[2+H+\xi B_1K\right]\frac{\Theta^2}{\nu_0}\frac{dU_1}{dX}=2l\,, \tag{9}$$

which is identical to the integrated momentum equation in incompressible flow, except for the term in ξB_1K, where K is given by Tani as $K=\frac{1}{\Theta}\int_0^\Delta\Phi\,dY$. The transformed momentum thickness, Θ, is defined as

$$\Theta=\int_0^\Delta\frac{U}{U_1}\left(1-\frac{U}{U_1}\right)dY\,, \tag{10}$$

H is given by the ratio, $H=\Delta^*/\Theta$, where Δ^*, the transformed displacement thickness, is

$$\Delta^*=\int_0^\Delta\left(1-\frac{U}{U_1}\right)dY\,, \tag{11}$$

while l is obtained from the equation,

$$l=\frac{\Theta}{U_1}\left[\frac{\partial U}{\partial Y}\right]_{Y=0}. \tag{12}$$

Now, for incompressible flow, Thwaites[2] expressed both H and l in the terms of the parameter m, where

$$m=-\frac{\Theta^2}{\nu_0}\frac{dU_1}{dX}\,. \tag{13}$$

The determination of H and l in terms of m will not be covered in detail, except to remark that both functions were evaluated by Thwaites independently of Eq. 9, and therefore his results are valid for the present analysis, even though Eq. 9 is not identical to the incompressible equation. By writing Eq. 9 in the form,

$$U_1\frac{d(\Theta^2/\nu_0)}{dX}=2\left\{\left[2+H(m)+\xi B_1K\right]m+l(m)\right\}=L(m), \tag{14}$$

it is seen that, for incompressible flow, with $B_1=0$, the right hand side, $L(m)$, is a function of m only. Both $H(m)$ and $l(m)$ have been plotted in Fig. 1, so that $L(m)$ may be obtained from Eq. 14. Thwaites approximated $L(m)$ by the linear function, $L(m)=0.45+6m$, and thus succeeded in integrating Eq. 14 directly to

$$\frac{\Theta^2}{\nu_0}=0.45\,U_1^{-6}\int_0^X U^5\,dX\,. \tag{15}$$

Equation 15 may be applied to compressible flow only for a Prandtl number equal to one, for which $\xi=0$. Some estimate of the error involved in using this equation for a Prandtl number of 0.72, corresponding to air at standard conditions, and for intermediate supersonic Mach numbers can be found from the consideration that, for these conditions, ξB_1K is approximately 10 per cent of $2+H(m)$. Such an error will be considered too large to ignore in the development of Eq. 15 from Eq. 14, and therefore Eq. 15 will not be used in the subsequent analysis.

A direct relation between Θ and X without the term in K may be obtained by integrating the product of the transformed momentum equation (8) with U across the boundary layer. By making use of the transformed continuity equation (6) and the relation given by Eq. 4, the result is

$$\frac{U_1}{\nu_0}\frac{d}{dX}(G^2\Theta^2)+\frac{6G^2\Theta^2}{\nu_0}(1+\tfrac{2}{3}\xi B_1)\frac{dU_1}{dX} =Q\left[1+J\left(\frac{1}{C_1}-1\right)\right], \tag{16}$$

where

$$G=\Theta^*/\Theta\,, \tag{17}$$

$$Q=\frac{4\Theta^*}{U_1^2}\int_0^\Delta\left(\frac{\partial U}{\partial Y}\right)^2dY\,, \tag{18}$$

and

$$J=\frac{\displaystyle\int_0^\Delta\left(\frac{1}{C}-1\right)\left(\frac{\partial U}{\partial Y}\right)^2dY}{\displaystyle\int_0^\Delta\left(\frac{1}{C_1}-1\right)\left(\frac{\partial U}{\partial Y}\right)^2dY}\,, \tag{19}$$

and where the quantity Θ^* is defined as

$$\Theta^*=\int_0^\Delta\frac{U}{U_1}\left(1-\frac{U^2}{U_1^2}\right)dY\,. \tag{20}$$

Tani evaluated G, Q, and J for both adverse and favorable pressure gradients by representing U and C as a power series in Y/Δ, and obtaining the constants in the series from the boundary conditions at $Y/\Delta=0$ and $Y/\Delta=1$. Tani chose to solve for the constants in terms of the function, $a'(X)=\partial(U/U_1)/\partial(Y/\Delta)$, rather than for m as is usually done.

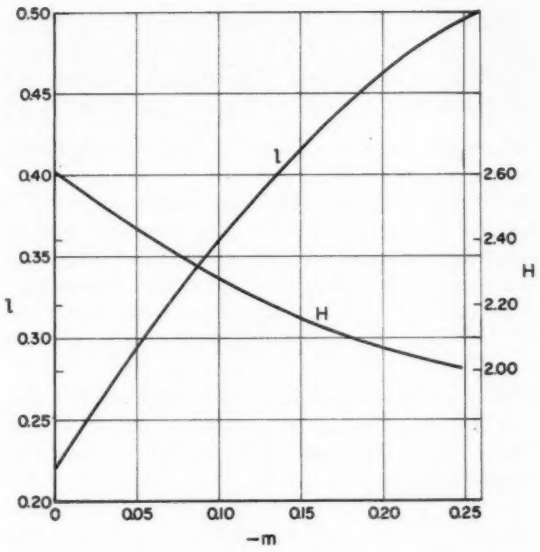


FIG. 1. FUNCTIONS $l(m)$ AND $H(m)$ FROM THWAITES

Momentum Thickness

The momentum thickness in the transformed flow may be obtained by integrating Eq. 16 with respect to X. The additional assumptions were made that the variables, Q/G^2 and J, could be replaced with the constants, 0.44 and 0.184, respectively, corresponding to their values for zero pressure gradient; and also that the variables C_1 and B_1 could be approximated by their free stream values, C_∞ and B_∞. Equation 16 then integrates to

$$\frac{\Theta^2}{\nu_0}=0.44\left[1+0.184\left(\frac{1}{C_\infty}-1\right)\right]\times \frac{\displaystyle\int_0^X U_1^{6(1+\frac{2}{3}\xi B_1)-1}\,dX}{U_1^{6(1+\frac{2}{3}\xi B_1)}}\,, \tag{21}$$

which is remarkably similar to Eq. 15 obtained by Thwaites. Although Eq. 21 in its present form could be used to calculate skin friction, it is more convenient to deal with quantities in the original compressible flow, rather than in the transformed flow. If we define the momentum thickness θ in the original flow as $\theta=\displaystyle\int_0^\delta\left(1-\frac{u}{u_1}\right)\frac{\rho u}{\rho_1 u_1}\,dy$, and express the other quantities in Eq. 21 in terms of the original flow variables, Eq. 21 becomes

$$\Omega^2=\frac{\sqrt{\gamma RT_\infty}}{\nu_0 C}\,\theta^2=0.44\left[1+0.184\left(\frac{1}{C_\infty}-1\right)\right]\left(\frac{T_w}{T_1}\right)^{\frac{2}{\gamma-1}[1-(1+\xi B_\infty)\sqrt{\sigma}\,(\gamma-1)]}\left(\frac{T_\infty}{T_1}\right)^{3(1+\frac{3}{5}\xi B_\infty)}\times$$

$$\left\{\frac{\int_0^{x/c} M_1{}^{(5+4\xi B_\infty)}\left(\frac{T_1}{T_\infty}\right)^{[(5/2)+2\xi B_\infty]}\left(\frac{T_1}{T_w}\right)^{\frac{2}{\gamma-1}\left[\frac{\gamma}{2}-(1+\xi B_\infty)(\gamma-1)\sqrt{\sigma}\right]}d\left(\frac{x}{c}\right)}{M_1{}^{6(1+\frac{3}{5}\xi B_\infty)}}\right\}, \tag{22}$$

where the local Mach number, M_1, rather than velocity u_1 has been used for convenience. The quantity Ω is referred to as the momentum thickness parameter. Its evaluation over the entire surface is a prerequisite for determining skin friction.

Skin Friction

Skin friction on a surface results from shear forces which are proportional to the velocity gradient in the boundary layer normal to the surface, the constant of proportionality being the absolute viscosity, μ. Thus, the shear force per unit area, τ, at any point along the surface may be expressed as $\tau=\mu_w\left[\frac{\partial u}{\partial y}\right]_w$. By making use of the definition of l (Eq. 12) and by writing τ in terms of variables in the original flow only, this becomes

$$\tau=\frac{\mu_0 l u_1}{\theta}\left(\frac{T_1}{T_w}\right), \tag{23}$$

where the reference temperature, T_0, and hence viscosity, μ_0, are equal to the values at the surface. Then, by defining the local skin friction coefficient, c_{f_1}, based on the local dynamic pressure, $\rho_1 u_1^2/2$, as $c_{f_1}=\frac{2\tau}{\rho_1 u_1^2}$, Eq. 23 becomes

$$c_{f_1}\sqrt{Re_1}=\frac{2l}{\Omega}\sqrt{\frac{(x/c)C_1}{M_1}}\left(\frac{T_1}{T_w}\right)^{\frac{\gamma-2}{2(\gamma-1)}}\left(\frac{T_1}{T_\infty}\right)^{-\frac{1}{4}}, \tag{24}$$

where the local Reynolds number $Re_1=\frac{u_1\,\rho_1\,x}{\mu_1}$, Ω is given by Eq. 22, and l may be found from Fig. 1 after m is known.

We may find m in terms of quantities in the transformed flow from Eq. 13. However, the calculation of skin friction is greatly simplified if m is also written as a function of the original flow variables. Equation 13 then becomes

$$m=-\Omega^2\left(\frac{T_w}{T_1}\right)^{\frac{2\gamma-3}{\gamma-1}}\left(\frac{T_1}{T_\infty}\right)^{\frac{1}{2}}\frac{dM_1/d(x/c)}{1+[(\gamma-1)/2]M_1^2}. \tag{25}$$

The local skin-friction coefficient on the surface may be obtained by evaluating Eqs. 22, 25, and 24 in that order, with l being found from Fig. 1. Before these equations may be evaluated, the Mach number M_1 and temperature T_1 in the local free stream must be obtained, the initial conditions being the free stream Mach number, M_∞, and temperature, T_∞, and the surface or wall temperature, T_w.

Up to this point, only the local skin friction at each point on the surface has been considered. The average skin friction coefficient, C_{F_∞}, based on the undisturbed free stream dynamic pressure, $\rho_\infty u_\infty^2/2$, is of practical importance, for it leads to the total viscous drag and may be readily compared with flat-plate values. By defining c_{f_∞} as the local skin-friction coefficient based on free stream dynamic pressure, it can be readily shown that

$$c_{f_\infty}\sqrt{Re_{\infty_c}}=\left(\frac{p_1}{p_\infty}\right)^{\frac{1}{2}}\left(\frac{M_1}{M_\infty}\right)^{3/2}\left(\frac{T_1}{T_\infty}\right)\left(\frac{T_\infty+216}{T_1+216}\right)^{\frac{1}{2}}\times\frac{c_{f_1}\sqrt{Re_1}}{(x/c)^{\frac{1}{2}}}, \tag{26}$$

where Re_{∞_c} is the free stream Reynolds number based on the chord length, c, and is defined as $Re_{\infty_c}=\frac{u_\infty\rho_\infty C}{\mu_\infty}$. The average skin-friction coefficient, C_{F_∞}, is related to c_{f_∞} by the expression,

$$C_{F_\infty}=\int_0^1 c_{f_\infty}d(x/c), \tag{27}$$

which could be used, together with Eq. 26, to obtain the average skin-friction coefficients on two-dimensional shapes, if it were not for the infinity which occurs in the integrand for $x/c=0$. This difficulty is overcome by assuming that the integrand varies as $(x/c)^{-\frac{1}{2}}$ in the interval $0\leq x/c\leq\eta$, η being a small positive number such that $0<\eta\ll 1$. (For the biconvex airfoil calculations, $\eta=0.01$.) Equation 27 is then replaced by the equivalent relation,

$$C_{F_\infty}=[2c_{f_\infty}]_\eta\,\eta+\int_\eta^1 c_{f_\infty}d(x/c). \tag{28}$$

Equations 26 and 28 enable $C_{F_\infty}\sqrt{Re_{\infty_c}}$ to be obtained, after $c_{f_1}\sqrt{Re_1}$ has been evaluated over the entire surface.

Results for Zero Pressure Gradient

For the particular case of zero pressure gradient, the skin friction and momentum thickness reduce to very simple expressions, and a comparison may be

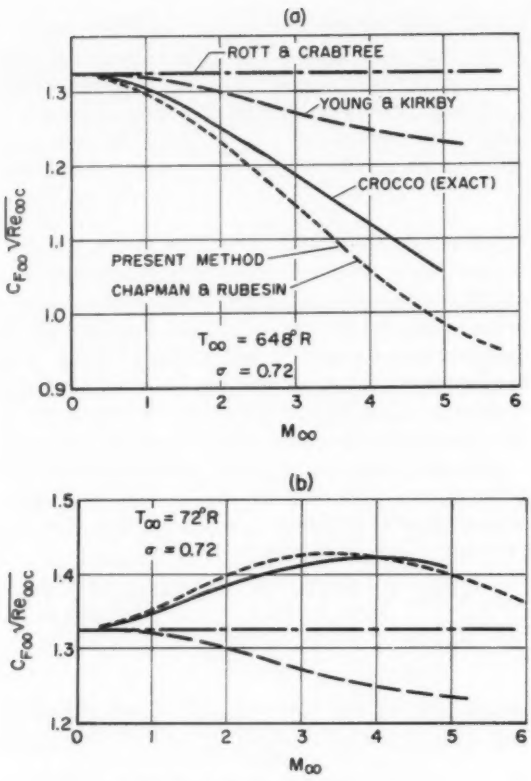


FIG. 2 FLAT-PLATE SKIN FRICTION

made with the results of other investigators. For this case, $m=0$ and $l=0.22$ and, therefore,

$$C_{F_\infty}=1.326\sqrt{\frac{C_\infty}{Re_{\infty_c}}}, \qquad (29)$$

which is identical with Eq. 58a of Chapman and Rubesin.[7] Equation 29 is compared in Figs. 2a and 2b, for $T_\infty=648°$ R and 72° R, respectively, with the method of Rott and Crabtree,[5] and with the calculations of Young and Kirkby.[10] The highly accurate but involved calculations of Crocco[11] have also been included for comparison. Equation 29 agrees reasonably well with Crocco's results. Rott and Crabtree's method, however, indicates a constant value of $C_{F_\infty}\sqrt{Re_{\infty_c}}$ with Mach number and temperature. This results from their use of the viscosity law, $\mu_0/\mu=T_0/T$, which is equivalent to the Chapman-Rubesin relation used in the present paper, with the constant $C_\infty=1$ for all conditions of temperature and Mach number. Thus, by setting $C_\infty=1$, they eliminated the Mach number- and temperature-dependent terms from their theory.

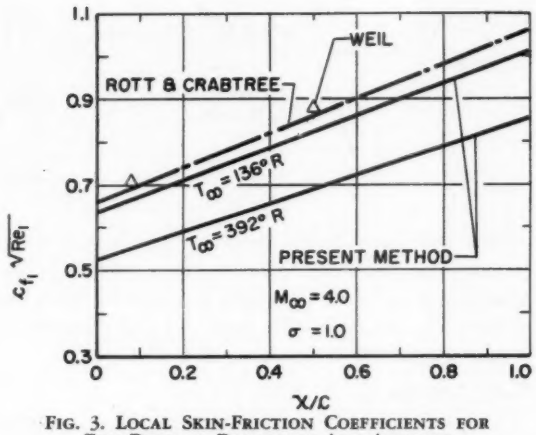


FIG. 3. LOCAL SKIN-FRICTION COEFFICIENTS FOR FIVE-PERCENT PARABOLIC ARC AIRFOILS

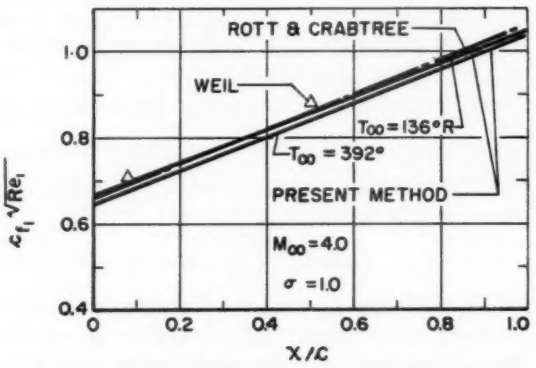


FIG. 4. LOCAL SKIN-FRICTION COEFFICIENTS WITH $C_\infty=1$ FOR FIVE-PERCENT PARABOLIC ARC AIRFOILS

Young and Kirkby's results, based on the viscosity law, $\mu_0/\mu=(T_0/T)^{8/9}$, indicate that $C_{F_\infty}\sqrt{Re_{\infty_c}}$ is independent of T_∞, and thus also fail to agree with Crocco's calculations. On the basis of these comparisons for zero pressure gradient, it was concluded that to correctly predict the variation of skin friction with Mach number and temperature, a viscosity law of the type defined by Eqs. 7a and 7b must be employed, even when the pressure gradient is not zero.

Airfoil Skin Friction

Skin friction calculations, by the present method, were then made for a 5-percent-thick parabolic arc airfoil section. The detailed calculations are given in Reference 8, but some of the results have been included in Figs. 3 through 6. The computations were carried out for $M_\infty=1.5$ and 4.0, $T_\infty=136°$ R and 392° R, and $\sigma=0.72$ and 1.0.

The variation of $c_{f_1}\sqrt{Re_1}$ with x/c for $M_\infty=4.0$ and $\sigma=1.0$ is indicated in Fig. 3 for both temperatures.

Calculations by the method of Rott and Crabtree and the two points by Weil[12] are also shown for comparison. Weil took $C_\infty = 1$, and it is therefore not surprising that his values agreed well with those by Rott and Crabtree's method.

By dividing the skin-friction coefficient by $\sqrt{C_\infty}$ the results of the present method become nearly coincident with those of Weil and with those obtained by Rott and Crabtree's method, as indicated in Fig. 4. Similar results were found for $M_\infty = 1.5$, and it therefore appears that, for favorable pressure gradients, the skin friction is nearly proportional to $\sqrt{C_\infty}$, in agreement with the results of Chapman and Rubesin for zero pressure gradient (Eq. 29).

Prandtl Number Effect

The foregoing calculations were then repeated for a Prandtl number of 0.72, corresponding to that for air in this temperature range. The change in skin friction was found to be very small (Table I). It was therefore concluded that, at least for the Prandtl number range from $0.72 \leqq \sigma \leqq 1.0$, this effect could be ignored. In addition, it is recommended that $\sigma = 1$ be used in the calculations rather than the actual value, because this simplifies the computations considerably.

TABLE I

AVERAGE SKIN FRICTION SUMMARY

Five-Percent Parabolic Arc Airfoil Section

M_∞	σ	Temperature		$C_{F_\infty}\sqrt{Re_{\infty_c}}$		
		Static	Total	Airfoil	Flat Plate	Increase (%)
1.5	1.0	392° R	.....	1.636	1.28	28
1.5	1.0	.....	569° R	1.636	1.28	28
1.5	0.72	392° R	.....	1.647	1.29	28
1.5	0.72	.....	569° R	1.647	1.29	28
4.0	1.0	392° R	.....	1.365	1.08	26
4.0	1.0	.....	569° R	1.635	1.25	31
4.0	0.72	392° R	.....	1.364	1.106	23
4.0	0.72	.....	569° R	1.633	1.266	29

Average Skin Friction

The ratio of the average airfoil skin-friction coefficient, C_{F_∞}, to that for a flat plate, $C_{F_{\infty FP}}$, at the same free stream Mach number and temperature, is plotted in Figs. 5a and 5b for $\sigma = 0.72$ and 1.0, respectively. The results of Young and Kirkby,[10] which, as mentioned previously, are based on the temperature-viscosity relation $\mu_0/\mu = (T_0/T)^{8/9}$, have been included in Fig. 5a for comparison, while those of Boison,[13] which are based on $\mu_0/\mu = T_0/T$, have been included in Fig. 5b for the same reason. Note

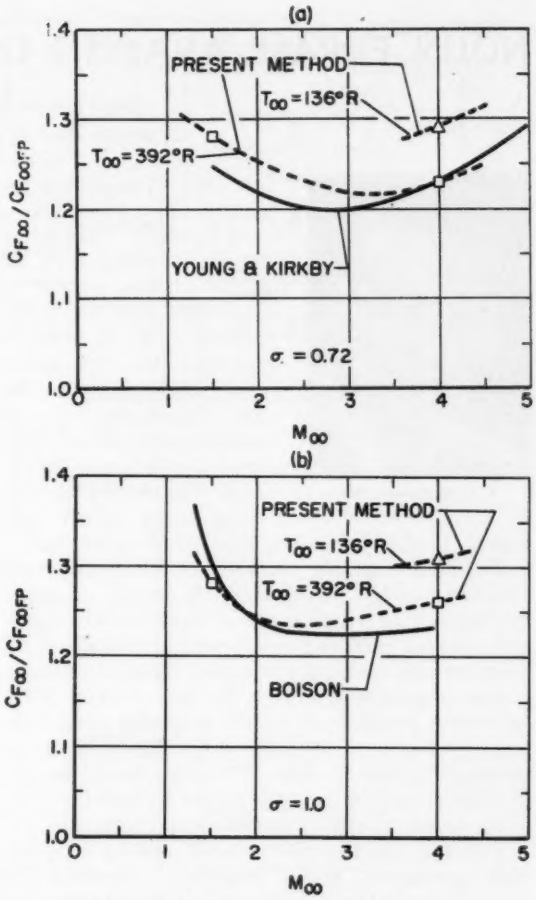


FIG 5. RATIO OF AIRFOIL TO FLAT-PLATE SKIN FRICTION

that, while the basic values of C_{F_∞} for Boison's and Young and Kirkby's calculations differ from those of the present method, the ratio $C_{F_\infty}/C_{F_{\infty FP}}$ is essentially the same.

A summary of the results obtained by the present method is given in Table I. Here it is shown that, for the cases considered, the average skin friction for the 5-percent airfoil section is 23 per cent to 31 per cent greater than for a flat plate.

Conclusion

A method of estimating compressible, laminar, skin friction on two-dimensional bodies with favorable pressure gradients has been presented. Equations 22, 24, and 25 are required to obtain local skin-friction coefficients, while Eqs. 26 and 28 are needed for average skin-friction coefficients.

(Continued on page 22)

NOUN PHRASE ANALYSIS USING THE IBM 650 COMPUTER

ARISTOTELIS D. STATHACOPOULOS
Graduate Student in Electrical Engineering

A. D. Stathacopoulos
Edye's Studio

At present, machine translation is accomplished by a word-for-word translating process, including some idiomatic phrases treated as independent units and translated as such. It is easily apparent that translation of this kind carries the desired information, but it requires special effort by the reader in order to be understood. The output of such a translating device will appear in the source-language word order and it will include multiple translations for words with several meanings in the target language. In other words, such an output is not in conventional English and requires polishing and rewriting before it can compare with the output of a good human translator.

Rearrangement of word order and selection of the contextual meaning of a word constitute two of the main problems of machine translation. A device which will overcome these difficulties should be able to perform certain logical operations which the mind of the human translator performs unconsciously.

This fact suggests the use of a modern computer with permanent stored program to analyze the source-language text and perform the changes required in the word-for-word equivalent target-language translation.

One particular problem is recognition of the word series consisting of article, adjectives, and final noun, called a noun phrase, which replaces the simple noun in more complex sentence structure. The following discussion describes a method for recognizing and analyzing a noun phrase in any sentence.

A noun phrase consists of articles, adjectives, and nouns. However, many of the constituents of a noun phrase may belong to more than one grammatical class. These words must be assigned to their correct class on the basis of context and translated accordingly. If this is to be done by a computer, each word must be accompanied by a class tag operated on by the computer program as it performs the desired logical operation while analyzing a group of words.

Binary numbers were selected as tags and the intersection sum of logical algebra was selected as the main operation for the program. In taking the intersection sum of two binary numbers the answer will have a *one* whenever we add two *ones* in the same column, while it will have zeros in any other case. The example below illustrates what is meant by the intersection sum.

```
  11001
        ← binary numbers
  10010
  -----
  10000 ← intersection sum
```

During the following discussion, by "addition" we mean the intersection sum.

The tags selected for this analysis are the following:

article	110001
adjective	100001
noun	100000

These tags have the following properties of logical algebra:

art.$\cap$art. = art. art.$\cap$adj. = adj. art.$\cap$noun = noun
adj.$\cap$adj. = adj. adj.$\cap$noun = noun
noun$\cap$noun = noun

The symbol $\cap$ indicates the intersection sum.

Each word will be represented by one tag for each grammatical class to which it belongs. Thus, words belonging to several grammatical classes will be represented by a series of tags in a predetermined order. For example, the word "book" will have the following series of tags:

100001 100000 111111

The tag 111111 is the tag assigned to any verb.

The following description outlines the essential features of the flow chart of the program as shown in Fig. 1, which provides for a noun phrase of up to six words.

The first tag of the first word is examined to determine whether it marks the beginning of a noun phrase or not. If not, a further analysis follows to determine the nature of the tag. However, if the first tag indicates the beginning of a noun phrase, the routine continues as follows: The first tags of the next five words are added consecutively to this tag, and after every addition, the result is tested to determine whether we have a noun phrase or not. If the first tags fail to indicate a noun phrase, the second tags are examined until a noun phrase tag is formed.

The following example of a three-word noun phrase will demonstrate the performance of the program:

The Australian team appeared.
110001 100001 100001 111111
000000 000000 100000 000000

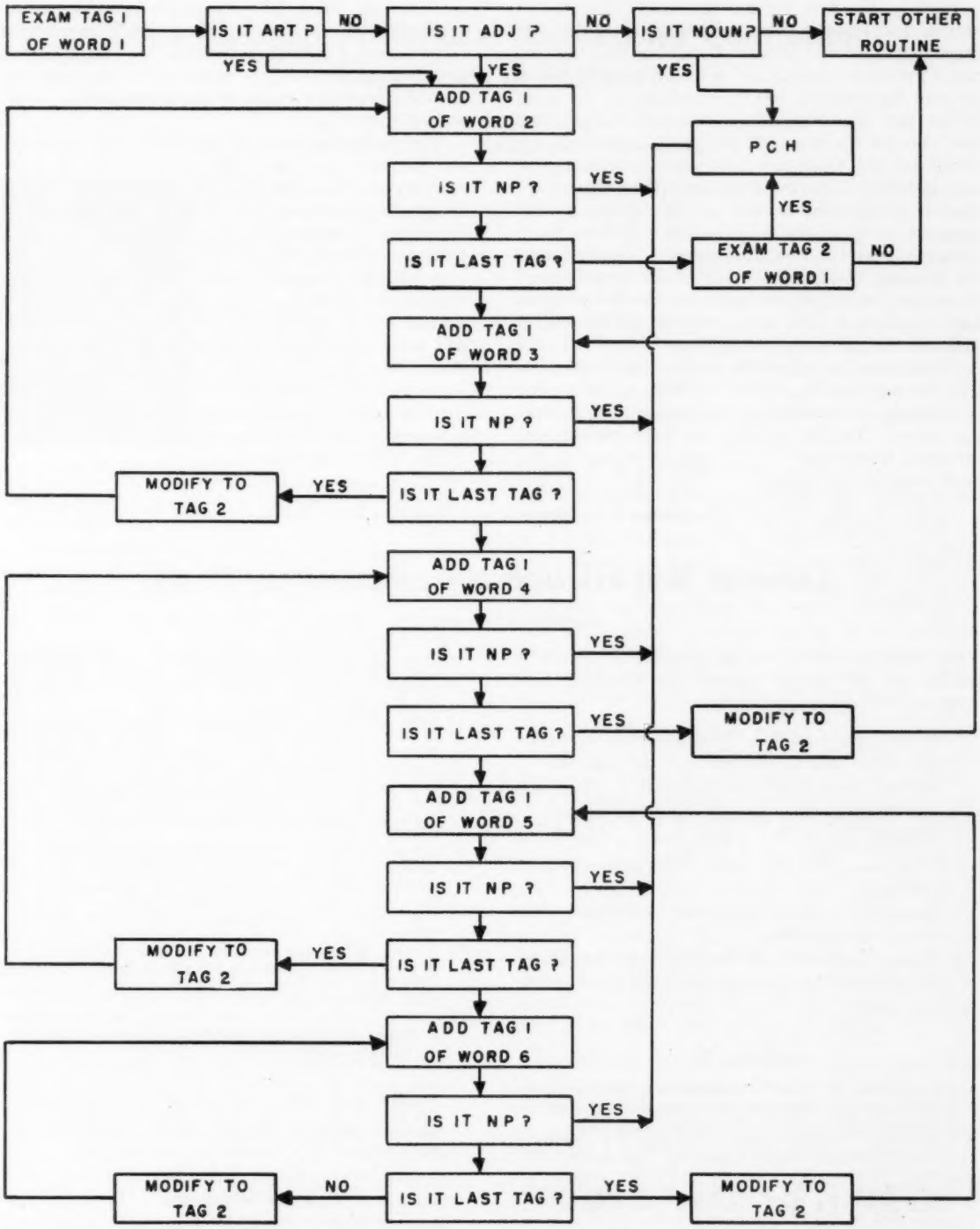


FIG. 1. FLOW CHART

Initially, the first tag for the word "the" is analyzed, and since it is an article tag, it marks the beginning of a noun phrase. Following this, the first tag of the word "Australian" is added to the previous tag and the result is 100001 which is not the noun phrase tag. Consecutively, in the same fashion, the first tags for the words "team" and "appeared" are added, but still these tags fail to form a noun phrase tag, 100000. Then the first tag of the word "appeared" is replaced by the second tag and a new intersection operation is performed. Still no noun phrase is obtained. Then "appeared" is dropped from the process. Next the first tag of the word "team" is replaced by its second tag and a new intersection sum is obtained. The final result is 100000, which indicates the end of the noun phrase.

At this point, a card will be punched by the machine with the tags 110001, 100001, 100000, which are the correct tags of the analyzed noun phrase "The Australian team." In this process the noun phrase was identified and "team" was found to belong to the noun class in this context.

Since the IBM 650 computer is a bi-quinary machine and is not capable of performing intersection summations, a special subroutine has been included in the program which is designed to transform the normal summation result to the corresponding intersection sum.

The program is able to determine the beginning and the end of the noun phrase and assign the correct grammatical class to each constituent. It also demonstrates the possibility of applying mathematical operations to sentence analysis and the use of a computer to carry out the process.

A complete program of this kind, constructed for analyzing sentences of the source language, could then be incorporated into a word-for-word translator and work on the tags of the source-language words. It could be expanded further to be capable of transforming the source-language analysis results to the corresponding target-language analysis and thus improve the accuracy, understandability, and readability of the final translation.

Laminar Skin Friction in Compressible Flows

(Continued from page 19)

Calculations carried out by this method for both airfoils and flat plates justified the following conclusions:

1. To accurately predict the variation of skin friction with Mach number and temperature, a viscosity law of the type employed by Chapman and Rubesin (Eqs. 7a and 7b) must be used.
2. Laminar skin friction is very nearly proportional to $\sqrt{C_\infty}$, for zero and favorable pressure gradients.
3. The effect of Prandtl number on laminar skin friction is negligible.
4. The ratio of airfoil to flat-plate skin friction is only slightly affected by the type of viscosity law used.

REFERENCES

1. Pohlhausen, K., "Zur Näherungsweisen Integration der Differential-gleichungen der Laminaren Reibungsschicht," *Zeitschrift f. agnew. Math und Mech.*, Bd. 1, 1921.
2. Thwaites, B., "Approximate Calculation of the Laminar Boundary Layer," *Aeron. Quarterly*, Vol. 1, Part III, Nov., 1949.
3. Illingworth, C. R., "Steady Flow in the Laminar Bound Layer of a Gas," *Proc. Roy. Soc.*, London, Vol. A199, 1949.
4. Stewartson, K., "Correlated Incompressible and Compressible Boundary Layers," *Proc. Roy. Soc.* London, Vol. A200, 1950.
5. Rott, Nicholas, and Crabtree, L. F., "Simplified Laminar Boundary Layer Calculations for Bodies of Revolution and for Yawed Wings," *Jour. Aeron. Sci.*, Vol. 19, No. 8 (Aug., 1952).
6. Rott, Nicholas, "Compressible Laminar Boundary Layer on a Heat Insulated Body," Readers Forum, *Jour. Aeron. Sci.*, Vol. 20, No. 1 (Jan., 1953).
7. Chapman, Dean R., and Rubesin, Morris W., "Temperature and Velocity Profiles in the Compressible Laminar Boundary Layer with Arbitrary Distribution of Surface Temperature," *Journ. Aeron. Sci.*, Vol. 16, No. 9 (Sept., 1949).
8. Levinsky, Ely S., "Approximate Compressible Laminar Skin Friction with Favorable Pressure Gradients," unpublished M.S. Thesis, University of Washington, 1956.
9. Tani, Itiro, "On the Approximate Solution of the Laminar Boundary Layer Equations," *Jour. Aeron. Sci.*, Vol. 21, No. 7 (July, 1954).
10. Young, A. D., and Kirkby, S., "The Profile Drag of Biconvex Wing Sections at Supersonic Speeds," *50 Jahre Grenzschichtforschung*, pp. 419-431, Friedr. Vieweg und Sohn, Braunschweig, 1955.
11. Crocco, L., "Lo Strato Linite Laminare nei Gas," *Monografic Scientifiche di Aeronautica*, Associazone Culturale Aeronautica, Rome, No. 3 (Dec., 1946).
12. Weil, H., "Effects of Pressure Gradient on Stability and Skin Friction in Boundary Layers in Compressible Fluids," *Journ. Aeron. Sci.*, Vol. 18, No. 5 (May, 1951).
13. Boison, J. C., "Mean Skin Friction Coefficients for Biconvex Airfoils in Supersonic Flow," Reader's Forum, *Journ. Aeron. Sci.*, Vol. 20, No. 11 (Nov., 1953).

ACKNOWLEDGMENT

The writer gratefully acknowledges the assistance of Professor Robert E. Street, of the Department of Aeronautical Engineering, University of Washington, in preparing this report.

Zone Melting: A Physical Method for Controlling Impurities in Metals

DOUGLAS H. POLONIS
Assistant Professor of Metallurgical Engineering

D. H. Polonis

Significance of Impurities

A thorough knowledge of the properties of pure metals is the ideal starting point for understanding the structure and behavior of complex alloys. Trace impurities which are inevitably present in metals will mask the physical properties of the ultrapure material. Consequently, very little is known about the properties of ultrapure metals at the present time, owing to the difficulties in achieving high purity.

The usual laboratory specimens which are specified as pure metals always contain trace impurities and are therefore not really pure in the true sense. Lead, for example, may be specified as 99.999+ per cent pure but detectable amounts of bismuth, copper, iron, and other metals will also be present. Until recently the attainable purity in the common metals has been limited by the efficiency of chemical extraction and refining processes. For many metals the purity obtainable falls far short of that for lead; for example, titanium can be prepared only 99.9+ per cent pure by thermal decomposition of titanium tetraiodide, and this is considered the highest grade titanium available. In order to increase the purity of metals far beyond that possible by chemical methods, the zone-melting technique has been developed and is becoming increasingly more important both in research and industry.

Nature of Impurities

Residual impurities in metals are traceable to several sources:

a) the ore from which the metal is obtained;
b) contaminants introduced during smelting, melting, casting;
c) residuals from refining;
d) contamination during heat treatment.

The increasing importance of the reactive metals in the aircraft industry and nuclear engineering has made item (d) more significant than in the past. The contamination of zirconium and titanium by small quantities of oxygen, nitrogen, carbon, and hydrogen has serious effects on physical properties.

Impurities may be present as dispersed inclusions such as oxides, sulfides, and silicates; as gases which cause internal cavities or blowholes; or as structural constitutents such as solid solutions or second phases, depending on the solubility of the impurities in the metal. Most minor trace elements are present in such small quantities that they form solid solutions with the metal; thus the impurity, in this case, is randomly distributed throughout the base metal or solvent on an atomic scale. These solid-solution impurities are of particular concern in the present discussion.

Effects of Minor Impurities

Trace impurities can have significant effects on the properties of metals. Foreign atoms in solid solution in a metal structure will cause some deviation of mechanical and electrical properties from those of the pure metal, particularly in view of the difference in atomic diameters and the probable valency difference of the solute. It is interesting at this point to consider several familiar examples where trace elements exert a definite influence which is explainable in terms of current theory of the metallic state.

The effects of minor impurities on mechanical properties is well illustrated by the case of iron containing carbon concentrations ranging from 10^{-3} to 10^{-1} weight per cent. Tensile testing reveals an upper and lower yield-point phenomenon which is familiar to most engineers in stress-strain relations for low-carbon steels. This anomalous yielding effect has been attributed to the clustering of carbon atoms in the form of an "atmosphere" around dislocations, thereby increasing the stress to initiate dislocation movements to produce slip.[1] Recent research has shown that some nonferrous metals, including molybdenum and cadmium, also exhibit upper and lower yield points when they contain interstitial solutes.

The development of semiconductors has presented some unusual metallurgical problems, particularly in connection with trace impurities in germanium and silicon and their effect on electrical properties. The distribution and types of small concentrations of foreign atoms, including phosphorus, arsenic, and antimony, in these semimetals has a great influence on their usefulness in electronics, notably as transistor materials. As a result of efforts to produce extremely pure silicon and germanium, the zone-refining process

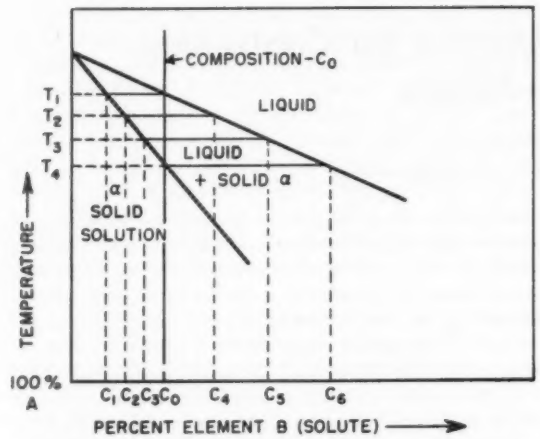


FIG. 1. **Partial phase diagram of a binary system, A-B, showing limited solid solubility and lowering of liquidus temperature with increasing solute concentration.**

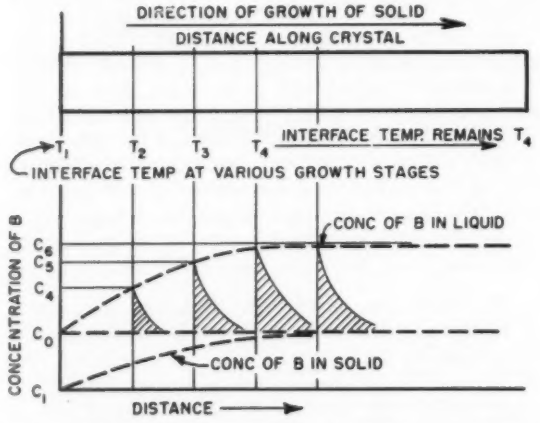


FIG. 2. **Schematic representation of interface temperature at successive stages of solidification and the corresponding solute concentration in solid and liquid at the interface.**

Note: **Shaded areas represent excess solute accumulation in the liquid at successive interface positions during the early stages of solidification.**

was developed at the metallurgical laboratories of the Bell Telephone Company.[2-5]

The zone-refining process is extremely simple in principle and, in this respect, is an elegant example of the application of fundamental principles in solving practical problems. The process is now being applied to the purification of many metals, primarily for engineering research purposes. This paper will analyze the principles of controlled segregation and zone refining.

Principles of Zone Melting

Zone melting is similar in principle to the fractional crystallization process which has long been used as a separation method by chemists. Briefly, the process involves purification by passing a molten zone along a bar of metal, thereby picking up impurities in the liquid and leaving behind a solid of lower impurity content. A basic requirement of this process is a higher solubility of impurities in the liquid phase than in the solid phase.

Details of zone melting can be most easily understood by referring to a portion of a simple phase diagram of the type shown in Fig. 1. In the system shown, the solute B lowers the freezing temperature of pure component A and causes solidification over a range of temperatures in accordance with the Phase Rule for a two-component system containing two condensed phases. This is a hypothetical case since most metals will contain more than one impurity, and consequently a binary diagram is an extreme simplification for actual conditions. It should be emphasized that freezing of a liquid metal is a dynamic process; therefore, interpretation of freezing processes by reference to a phase diagram provides only an approximate picture of the actual conditions, even during low transformation rates. Nevertheless, it is possible to estimate conditions at a solid-liquid interface during solidification if one assumes that the solute concentrations at the interface for any temperature in the α+liquid region are approximately those predicted by the phase diagram.

Consider the directional solidification of a long rod-like charge of molten alloy of composition C_0 in which the temperature is a minimum at the extreme left and increases uniformly along the charge. The first solid forms when the cool end of the charge becomes lower than temperature T_1 and will have an approximate composition C_1 which is considerably lower than the average solute concentration of the original liquid. The formation of solid phase establishes a solid-liquid interface which will move along the charge as solidification proceeds. This is shown schematically in Fig. 2, which illustrates the ideal case for uniform longitudinal heat flow. The solute depletion in the first solid to form provides the basis of purification. As freezing continues, the temperature at the interface must decrease and, accordingly, the composition of the solid phase forming at any temperature will be given approximately by the temperature intercept on the solidus line; i.e., the composition follows: C_1, C_2, C_3, C_0 between temperatures T_1 and T_4. Eventually, composition of the solid at the interface reaches a maximum of C_0 when the temperature reaches T_4. While the composition of the solid changes as described above, a simultaneous build-up of solute in the liquid takes place at the interface, where the composition follows the liquidus line between temperatures T_1 and T_4. It is seen that the maximum solute concentration in the liquid

reaches a limiting value C_e; this concentration build-up in the liquid compensates for the depletion in the solid. If the molten charge is long enough so that a portion of the charge remains molten when the interface temperature reaches T_4, then a condition of "kinetic equilibrium" is reached wherein the solute concentration at the interface in both solid and liquid (C_0 in solid and C_e in liquid) and the interface temperature remain approximately constant as the interface moves along the charge. The last liquid to solidify will contain the excess solute which was removed from the first solid to form. Consequently, the impurity concentration is built up in the final solid to freeze at the end of the rod. The concentration distribution of impurity along the rod after solidification is completed is shown in Fig. 3. It is important to note the following facts regarding this process:

1) Purification is effective only during the early stages of solidification, when the interface temperature is changing between T_1 and T_4. During this stage the solid forming has a lower impurity content than the initial liquid, and some purification is achieved. This is called the transitional stage as opposed to the steady-state stage, which is reached at temperature T_4 and continues until freezing is completed.

2) Diffusion is occurring in both the solid and liquid phases at all times. The amount of solute redistribution in the solid after freezing is negligible in practice. The transfer of solute across the interface and diffusion through the solid to level the concentration gradient is so slow that it does not alter the solute distribution significantly during the time required to solidify the charge. There will be appreciable redistribution of solute in the liquid due to diffusion and convection which result from the high atomic mobility in the liquid.

3) The process described thus far is merely an idealized picture of solid-solution segregation commonly referred to as "coring" and found in many commercial alloys in the as-cast condition.

An important factor which indicates the efficiency of segregation in solid-solution freezing is the separation of solidus and liquidus lines. This can be expressed as the Distribution Coefficient k at any temperature below the melting point for Fig. 1. Thus,

$$k = \frac{\text{Solute concentration in solid}}{\text{Solute concentration in liquid}} = \frac{C_S}{C_L}$$

At temp T_1, $k = \frac{C_1}{C_0}$. At temp T_2, $k = \frac{C_2}{C_4}$.

Reference to Fig. 1 reveals that the first solid to form has a concentration kC_0. Consequently, in order to have a purification process, it is necessary that $k<1$. The purification effect ends when the solid

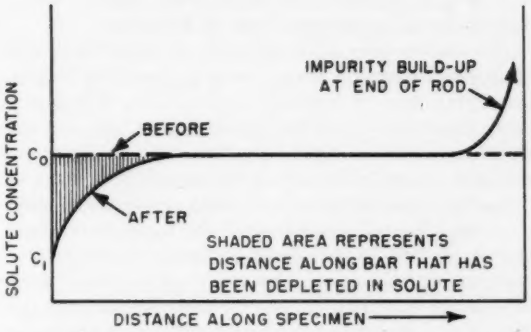


FIG. 3. Approximate concentration distribution along a rod after directional solidification.

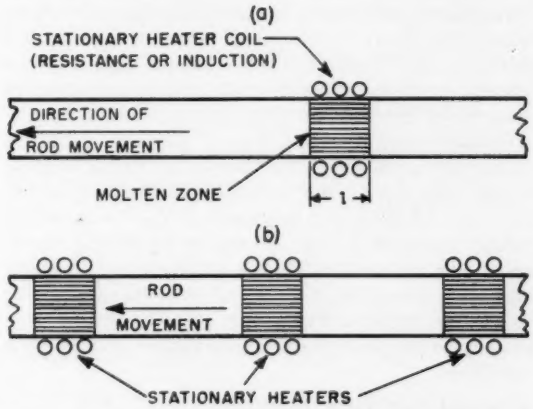


FIG. 4. Schematic diagram of zone-refining technique. Rod specimen is drawn through heater coil and molten zone moves along the specimen.
(a) Single pass setup. (b) Multiple zone-refining setup, with three molten zones passing simultaneously.

reaches concentration C_0, in which case the corresponding concentration at the interface in the liquid has a value C_0/k.

Single Pass Zone Refining

The purification effect of the transitional solidification stage previously described can be successfully achieved by passing a molten zone along the specimen. In this method only a fraction of the length of a specimen is molten at any time, so the impurity content builds up in the molten zone and is moved to the end of the specimen. The impurity content is thus concentrated in the last liquid to freeze. The concentration of impurity will build up in the liquid, with normal diffusion processes tending to level concentration gradients in the liquid zone owing to relatively high atomic mobility in the liquid phase. The mobile molten zone can be most simply controlled by drawing a rod-like specimen through an induction coil or resistance furnace as shown in Fig. 4. In this

way only a localized portion of the charge will be heated above the melting point at any time.

If a molten zone of length l is started at one end of a metal bar and passed slowly along its length, the segregation of impurity is similar to that previously described for directional solidification. As the zone moves to the right, the first solid to freeze as the zone moves will have a low solute content, and the solute concentration will build up in the molten zone until the solute capacity of the liquid is reached. It is important to note the significance of the rate of movement of the zone. The ability of the liquid zone to contain impurity atoms depends largely on the diffusion of impurities through the liquid if the maximum capacity of the liquid for impurity is to be utilized. Rapid zone movement should be avoided for efficient purification of the solid, since some time should be allowed for redistribution of the accumulated impurity within the molten zone. This does not present a serious experimental problem since normal convection effects in the liquid cause fairly rapid distribution of impurity in the zone. After the zone has picked up its capacity of impurity, the solid freezing at any time during the remainder of the process will have an approximate composition C_0. The excess solute deposited in the final region to solidify corresponds to the amount which was removed during the early stages and swept along in the molten zone. The solute distribution curve of Fig. 3 also applies approximately to the passage of a single molten zone along a specimen.

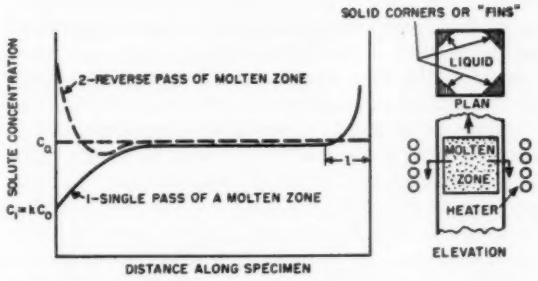
Repeated Pass Zone Refining

In order to achieve a high degree of purity along most of the length of a specimen, it is necessary to repeatedly pass molten zones along the specimen. Passage of successive molten zones progressively increases the purity of the initial regions at the starting end of the charge and extends the transitional, or purification, stage further along the bar with successive passes. Since with each successive pass there is less impurity to pick up in the initial end of the bar, the zone moves farther along the bar with each successive pass before reaching its capacity of impurity. Consequently, after a large number of passes in one direction, the impurity is concentrated at the final end of the specimen. The degree of impurity obtained throughout most of the bar is limited by the number of molten passes. The smaller the value of $k=(C_S/C_L)$, the more efficient the process per pass.

It should be noted that repeated pass zone melting can result in tapering a rod-shaped specimen. A slight taper may result even from a single pass and this effect will be quite noticeable after several passes.

Producing Homogeneous Solute Distribution

In some cases it has been desired to produce metal specimens containing dilute alloy concentrations in



(Left): **FIG. 5. Solute distribution along a specimen after single pass and reverse pass of molten zone.**

(Right): **FIG. 6. "Cage" zone-refining technique which eliminates ceramic containers.**

Note: **Bar may be of polygonal cross section other than rectangular.**

which the solute is uniformly distributed throughout. This is difficult with normal casting because of solid-solution segregation. Uniform solute distribution can be achieved quite simply by the following three methods:

a) *Directional solidification:* This method will produce a solute distribution similar to that shown in Fig. 3. A fairly uniform composition specimen can be obtained by removing the two ends of the specimen—one which is depleted in solute and the other enriched in solute.

b) *Reverse pass zone melting:* Re-zone melting in the reverse direction after the initial pass will improve the uniformity of solute distribution. The effects of reverse passing are schematically illustrated in Fig. 5.

c) *Use of a starting charge:* By building up the solute concentration in the initial region of the bar, the initial liquid zone which forms in this region can be made to have its maximum solute content and will therefore not pick up further impurity as it is passed along the bar. The optimum starting concentration will be C_0/k corresponding to the composition C_6 in Fig. 1. Since the liquid zone will then have its maximum capacity of solute, the result will be a composition approximating C_0 along the entire length of a metal bar, with the exception of the very last portion to freeze.

Application of Zone Refining

This paper has emphasized the basic principles involved in this relatively new and fascinating process for producing "super-pure" metals. The experimental techniques are relatively simple for the common metals with moderate melting temperatures. Zone-melting has been successfully applied to germanium and silicon for semiconductors. Work has been carried out on lead, tin, antimony, and other metals with astounding success. The techniques involve a protective atmosphere for the metal and a suitable

boat or crucible which will not introduce contamination; for example, graphite boats are frequently used for germanium, lead, and tin refining.

Remarkable success has been reported in removing arsenic from antimony by zone refining. After ten successive molten zone passes, the only impurities detectable by mass spectroscopy were zinc and arsenic in the order of one part in ten million.

Some modifications of zone refining have been attempted for purifying reactive metals with high melting temperature, but this work is still in the early experimental stages. An example is the "cage" zone refining technique[6] which has been applied with some success to titanium. Molten titanium reacts viciously with common refractory materials thus preventing the use of conventional zone-melting techniques for its purification. In cage-zone refining, a suspended solid titanium specimen is moved vertically through an induction heating coil without contacting refractory. If the cross section of the titanium bar is rectangular, then only the interior of the bar over the short length within the induction field may become molten; the corners of the specimen remain solid, owing to rapid local heat dissipation, to form a cage that confines the liquid. This cage effect is shown in Fig. 6. Surface tension effects and the levitating effect of the electromagnetic field, which depends on the design of the induction coil, prevent the liquid from spilling out.[7] Movement of the molten zone results in the purification effect. The fins or corners of the bar are not purified since they remain solid, but can be cut away leaving the purified core. Single crystals of titanium have not been prepared by this technique, but recent purification results appear very promising.[8]

In general, zone-melting techniques provide powerful tools for metallurgical research. Of particular interest in the future will be the development of zone melting to purify many of the higher melting temperature metals. Also, expanded research will undoubtedly be conducted on zone-refined metals to determine the true properties of pure metals and the relationship between minor impurities and structural imperfections.

REFERENCES

1. Cottrell, A. H., *Dislocations and Plastic Flow in Crystals*, Oxford Uni. Press, 1953.
2. Pfann, W. G., "Principles of Zone Melting," *Trans. A.I.M.E.*, **194**, 747 (1952).
3. Schumacher, E. E., "Ultra Pure Metals Produced by Zone Melting Technique," *Jour. of Metals*, Nov., 1953.
4. Pfann, W. G., "Redistribution of Solutes by Formation and Solidification of a Molten Zone," *Trans. A.I.M.E.*, **200**, 294 (1954).
5. Tanenbaum, M., Goss, A. J., and Pfann, W. G., "Purification of Antimony and Tin by a New Method of Zone Refining," *Trans. A.I.M.E.*, **200**, 762 (1954).
6. Brace, P. H., Cochardt, A. W., and Comenetz, G., "Cage Zone Refining," *Review of Scient. Inst.*, Vol. 26 (1955), p. 303.
7. Polonis, D. H., Butters, R. G., and Parr, J. G., "Levitation Melting Titanium and Titanium Alloys," *Research* (London), 7, 512, (1954).
8. Comenetz, G., private communication.

Recognizing and Developing Creative Potential

(Continued from page 1)

be developed through experience and exercise. It has been said that nothing breeds success like success, and this is probably true, but the corollary that failure breeds failure need not be true. If, through continued application, failures can be corrected, high orders of self-confidence can be developed. Actually, the fear of making a mistake is a very devastating emotional block to creative activity. People should realize that progress is made through failure as well as through success. I have had better success in training creative designers by helping them develop this spirit of self-confidence than I have in imbuing them with special design techniques or tricks of the trade.

The last emotional attribute I will mention here is drive. Many people have indicated that they feel that this is the prime requisite of all creative workers....

Developing Creative Potential: The steps in the creative process have been listed as: question, observe, associate, and predict. And that last term, predict, covers a multitude of concepts, for it includes not only communicating your ideas to someone else, but communicating them in a tangible form which requires initiative, daring, self-confidence, and tremendous drive. Many of the best ideas had to be sold under extremely unfavorable conditions, and this is no easy job. To do it requires faith, not only in yourself, but also in the worthwhileness of your project. Keen analysis, keen judgment, help provide and bolster this faith, and so they are as essential to this phase as they are to the idea-getting phase....

Helping Others Develop Creative Potential: What can you do to help others realize their creative potential and exercise it for the good of man? The management of creative personnel is a very complex problem, and, unfortunately, little concrete evidence is available to guide us. I do believe, however, that management has five responsibilities to the creative person. It must stimulate, encourage, assist, recognize and reward the creative person in order that he be most productive, though it is difficult to lay down any general rules for doing any one of these things and having them equally effective on all people. People vary considerably, and what stimulates one person may over-stimulate the next....

Some people can work to stringent deadlines; other people narrow their fields to such a degree that only commonplace results are obtained. One general rule might be made, however, and that is to try to establish a permissive atmosphere, one that establishes psychological safety and psychological freedom for the workers in that area.

An Aid in the Preliminary Design of Prestressed Concrete Beams

JACK R. CLANTON

Associate Professor of Civil Engineering

J. R. Clanton

Introduction

In a prestressed concrete beam it is desirable to have the full allowed unit compressive stress available to resist the bending moment from the superimposed load and at the same time to have little or no tensile stress on the beam section. In order to fulfill this ideal condition, the combination of the effect of prestress and the effect of the beam weight should give zero stress (or a comparatively small tensile stress) at the top of the beam and full allowed unit compression stress at the bottom.

If the resultant of the initial prestress force (F_i) is at the lower kern limit, its effect is to give zero stress at the top of the section, that is, $\frac{F_i}{A}-\frac{F_i e_1 y_t}{I}=0$, from which $e_1=\frac{r^2}{y_t}$, as shown in Fig. 1. The value of F_i, as given in Fig. 1, to make the unit stress at the bottom of the beam equal the full allowed value, is obtained from the allowed unit concrete stress at time of prestress: $f_{ci}=\frac{F_i}{A}+\frac{F_i e_1 y_b}{Ar^2}$, when r^2/y_t is substituted for e_1. The effect of bending moment (M_G) due to the beam weight is counteracted by lowering the resultant of the tensioning force an amount $e_2=M_G/F_i$, as shown in Fig. 2. Then the effect of initial prestress alone, and the location of the resultant force to give this effect, will be as shown in Fig. 3. It follows that the combined effect of initial prestress and bending due to beam weight will be as shown in Fig. 4.

Subsequent to the prestressing, there will be a decrease in F_i because of shortening of the beam due to concrete shrinkage, creep, and, in the case of pretensioned beams, elastic deformation. Creep of the highly stressed steel may also decrease the value of F_i so that the final prestress force (F) may be 80 to 85 per cent of F_i; therefore, a good estimate of the required section modulus may be made by multiplying the superimposed-load bending moment by a factor of 1.18 to 1.25 and dividing by f_{ci}. A section having a modulus at least as large as the computed value may then be selected from the chart, Figs. 5a and b. Finally, the selected section should be checked by computing the maximum stress intensities.

The procedure discussed above is for the preliminary design at the section of maximum moment. For a complete design, other sections may need investigation, and diagonal tension and anchorage conditions must be checked. In some cases, it may be more economical to revise the section to one that is unsymmetrical. In all cases of post-tensioned beams, the cables must be draped to give an eccentricity of no more than e_1 at the end, if tensile stresses are not to exist at this point. In the case of pretensioned beams, if the effect of beam weight is to be canceled, there must be a variable beam depth, or a camber in the beam, so that the eccentricity can vary from e at the point of maximum moment to e_1 or less at the ends.

The proportions of the beam sections, as given in Figs. 5a and b, follow in general the recommendations made by Guyon[1] as follows:

Web thickness: $h<30$ in.: $b'=d/7$ to $d/8$
$h>30$ in.: $b'=4$ in. $+d/40$

Flange thickness t: approximately $h/8$ and not less than $0.1h$ for h of 5 ft to 8 ft.

Width of flange b: not to exceed $0.8h$ and not less than $0.4h$.

Three-inch fillets were used in all sections for ease in preparing the chart, although in final design it will usually be desirable to detail them to a smaller slope carried out to the full flange width. In most of the sections a change in the fillet will not appreciably change the section properties; nevertheless, for a final design the computations should be based on the new section properties.

---

[1] Y. Guyon, *Prestressed Concrete*, New York, John Wiley & Sons, Inc., 1953.

Preliminary Design Example

Select a beam section to carry a 50-k load applied at the center of a 40-ft simple span.

$f'_c = 5{,}000$ psi; $f_c = 0.4\, f'_c = 2{,}000$ psi;

$f'_{ci} = 4{,}000$ psi; $f_{ci} = 0.55\, f'_{ci} = 2{,}200$ psi;

$$M_s = \frac{50\,(40)}{4} = 500 \text{ k-ft.}$$

Assuming 15 per cent loss of prestress:

$$\text{Required } Z = \frac{M(1.18)12}{f_{ci}} = \frac{500(1.18)12}{2.2} = 3{,}220 \text{ in.}^3$$

From the appropriate part of the chart, the lightest section with the required flexural resistance has the following properties:

$h = 38$ in., $b = 18$ in., $b' = 5$ in., $t = 5$ in.,
$A = 338$ sq in., $Z = 3{,}240$;

$$W_G = (338/144)\ (0.15) = 0.352 \text{ k/ft};$$

$$M_G = \frac{0.352(40)^2}{8} = 70.4 \text{ k-ft};$$

$$F_i = \frac{Af_{ci}}{1+(y_b/y_t)} = \frac{338(2.2)}{2} = 372 \text{ k};$$

$$F = 0.85\, F_i = 316 \text{ k};$$

$$e = \frac{r^2}{y_t} + \frac{M_G}{F_i} = \frac{61{,}569}{338(19)} + \frac{70.4(12)}{372}$$
$$= 9.59 + 2.27 = 11.86 \text{ in.}$$

Check stress condition at time of prestress:

$$f_t = -\frac{372}{338} + \frac{372(11.86)}{3{,}240} - \frac{70.4(12)}{3{,}240}$$
$$= -1.10 + 1.36 - 0.26 = 0$$

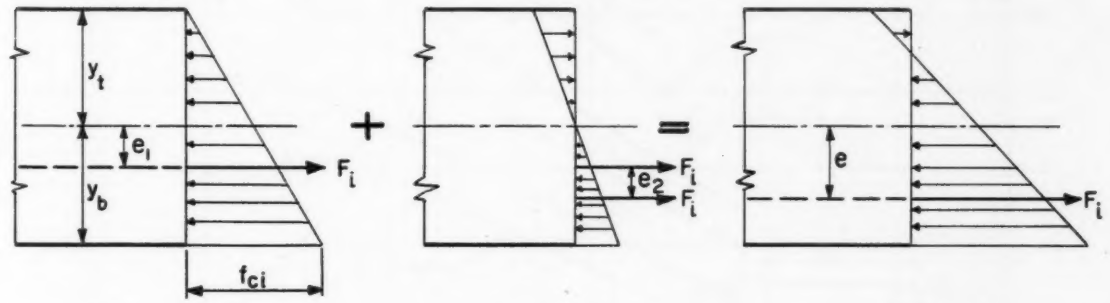
$$f_b = -1.10 - 1.36 + 0.26 = -2.20 \text{ k/sq in.}$$

Check stress condition at time of superimposing loads:

$$f_t = -\frac{316}{338} + \frac{316(11.86)}{3{,}240} - 0.26 - \frac{500(12)}{3{,}240}$$
$$= -0.94 + 1.16 - 0.26 - 1.85 = -1.89 \text{ k/sq in.}$$

$$f_b = -0.94 - 1.16 + 0.26 + 1.85 = +0.01 \text{ k/sq in.}$$

The selected section is satisfactory, in that the compression at the top of the beam is below the allowed value of 2.0 k/sq in. and the small amount of tension at the bottom of the beam is hardly of significance.



$= \frac{r^2}{y_t}$; $F_i = \frac{Af_{ci}}{1+(y_f/y_t)}$

FIG. 1. Prestress Force at Kern Limit

$M_G = F_i e_2$; $e^2 = M_G/F_i$

FIG. 2. Counteracting Beam-Weight Moment

$e = e_1 + e_2$
$e = (r^2/y_t) + (M_G/F_i)$

FIG. 3. Location of Prestress Force

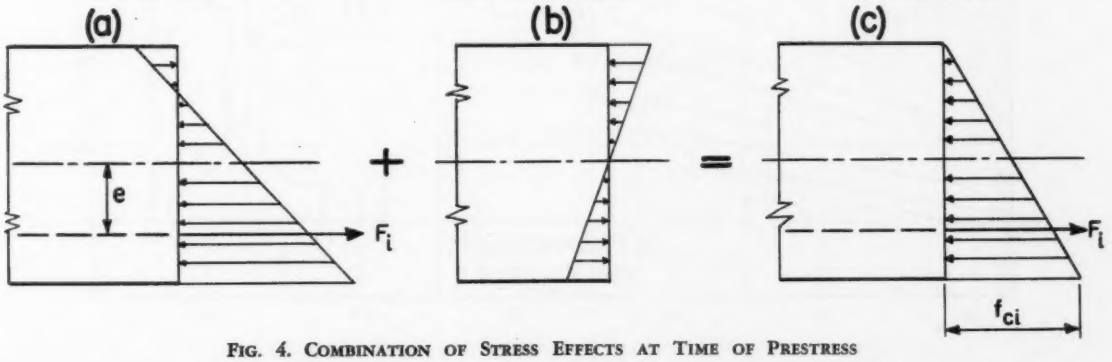


FIG. 4. COMBINATION OF STRESS EFFECTS AT TIME OF PRESTRESS

(a) Effect of prestress force. (b) Effect of beam weight. (c) Resultant effect.

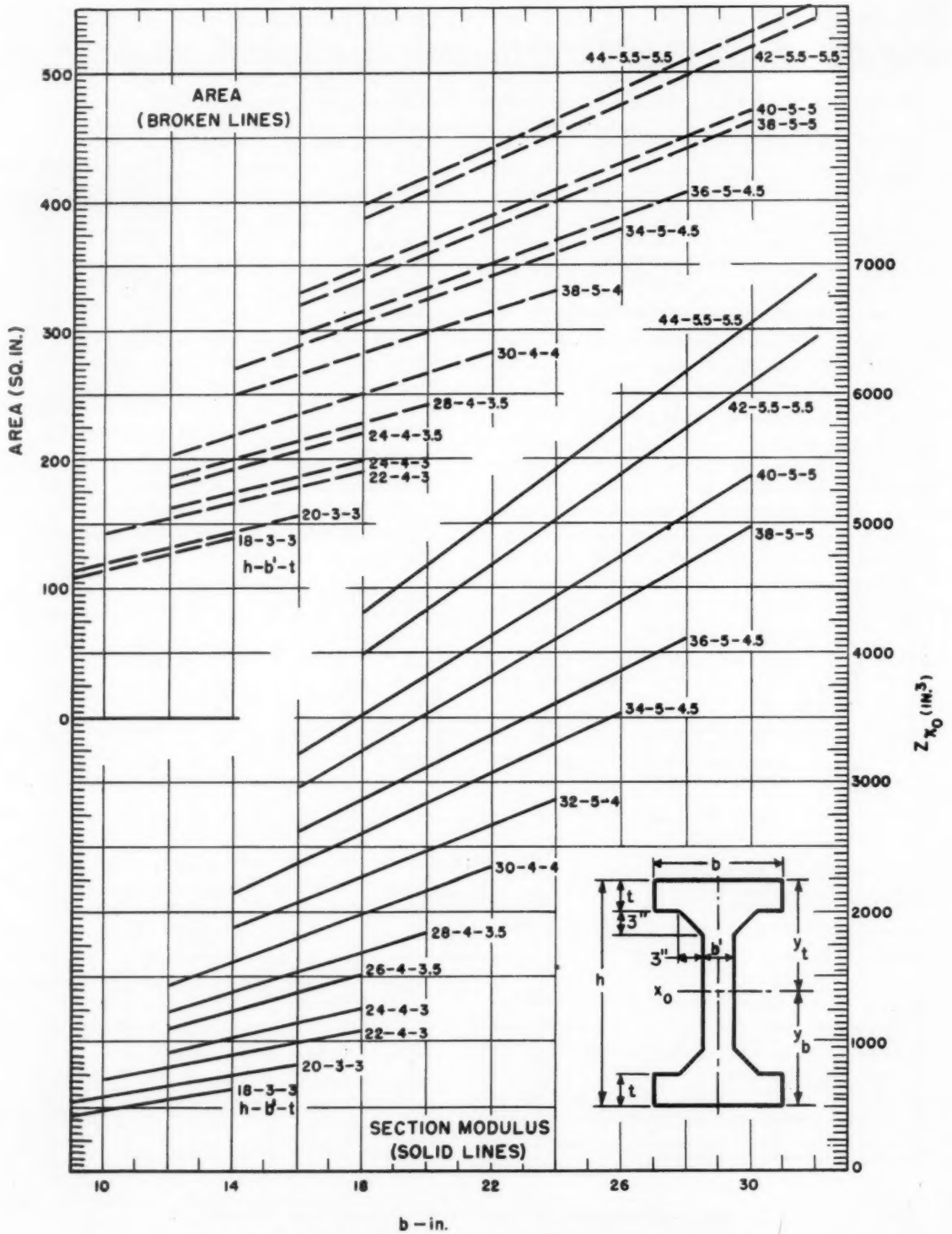


FIG. 5a. SECTION MODULUS AND AREA FOR SYMMETRICAL I SECTIONS

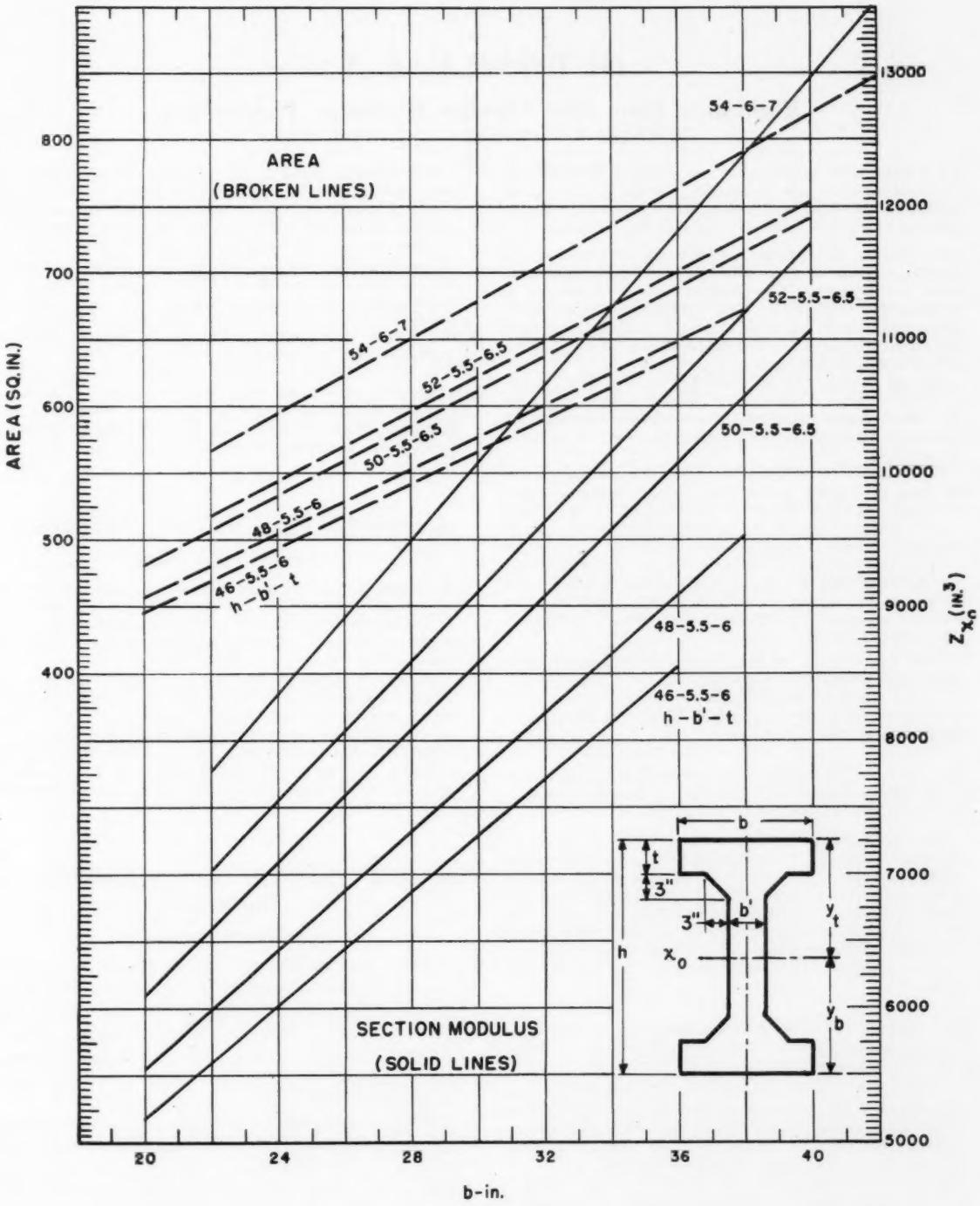


FIG. 5b. SECTION MODULUS AND AREA FOR SYMMETRICAL I SECTIONS *(continued)*

OUTSIDE U. S. A.

Highlights from Our Foreign Exchange Publications

AN APPROXIMATE SOLUTION OF THE PRANDTL EQUATION OF FLUID MOTION IN THE BOUNDARY LAYER. P. Szymanski, *Archiwum Mechaniki Stosowanej*, VII-1, 1955. Polish. (English abstract——more detailed than the following).

A complete approximate solution of the well known Prandtl equation of fluid motion in the boundary layer is given in this paper. The proposed method of solution is based on that of Pohlhausen, the procedure, however, being considerably simplified. This eliminates tedious calculations and permits one to obtain a complete solution in the form of general equations which are easy to handle.

Starting from the Prandtl equation we determine the approximate velocity profile of the boundary layer by means of a fourth degree polynomial, the coefficients of which are determined from the boundary conditions and the equation of continuity.

Thus, starting from the equations of the boundary layer and from the boundary conditions alone, we obtain the same expressions as those obtained by Loyciansky (Loitsianskii).

Next, the thickness of the boundary layer is found from the Kàrmàn integral condition, thus permitting us to obtain a differential equation, which is analogous to that of Pohlhausen. The integral λ (s) of this equation constitutes the solution of the problem. It vanishes for $s=s_m$, for which the potential velocity $U(s)$ reaches its maximum, and has a finite value λ_0 for $s=0$.

From this solution, we obtain the value $s=s_d$, for which value the boundary layer separates.

In spite of certain simplifications, the results of numerical calculations are in good agreement with those obtained by Pohlhausen by exact integration.

ON THE HEAT DISSIPATION OF 4-STROKE CYCLE ENGINES. Osamu Hiras, *Report of the Institute of Industrial Science*, Uni. of Tokyo, August, 1954. English.

The author found that the amount of heat dissipated to the water jacket of a gasoline engine increased whenever operating conditions or design were changed in such manner as to increase the temperature of the combustion gases, the time of contact of the hot gas with the cylinder walls, the area of the cylinder walls or the heat transfer coefficient between the walls and the hot gas. A theoretical analysis supplied an equation which correlated with the experimental results.

NEW ELECTROLYTIC ETCHANTS FOR STAINLESS STEELS CONTAINING SIGMA PHASE: I. Bertetti, *Metallurgia Italiana*, Vol. 68, No. 7, 1956, Italian.

A series of new electrolytic etchants for differentiating sigma phase from the matrix in stainless steels of both the ferritic and austenitic types have been studied. For ferritic stainless, the optimum concentrations and times of etching for solutions at room temperature are as follows:

a) 5% ammonium persulphate	5 seconds
b) saturated solution of sodium perborate	1 minute
c) 2% sodium periodate	10 seconds
d) 5% sodium bromate	5 seconds

All of these solutions darken and outline sigma phase.

For austenitic stainless, the periodate and perborate solutions used above do not give sufficient distinction between the sigma phase and any carbide phase present. The persulfate solution attacks and colors both the sigma and carbide. It is possible, however, to recognize the sigma both by the shape of the crystallites and the fact that they appear blue under the oil immersion objective. This coloration is not given to the carbide. A two-minute etching time is used. With the bromate solution, greater attack on the sigma phase than on the carbide is observed, permitting easy identification. An etching time of two minutes and 30 seconds is used.

In all studies an anode-cathode distance of 8 mm was used. The current was 0.5A and voltage 6V. (Specimen size and anode material are not given).——**Abstracted and translated from the Italian by E. C. ROBERTS, *Associate Professor of Metallurgical Engineering.***

CONSIDER ENGINEERING TEACHING

(Continued from page 2)

be earned through summer employment with industry or on research contracts. These engagements are now the rule rather than the exception. Also, as one acquires a reputation in his field, his services are sought on a consulting basis.

There is no question but what one must be prepared to accept some sacrifice in salary by selecting a teaching career rather than an industrial career—but the sacrifice is offset by the compensating factors noted in the preceding paragraphs. Happiness and satisfaction are somewhat intangible qualities, but the wise man will give them a high evaluation on his balance sheet. Not many college teachers have ulcers!

DIGITAL COMPUTER FOR HYDRO-STORAGE PROBLEMS

(Continued from page 12)

with a certain amount of programming experience. To accomplish most efficiently the solution of large system studies, such as that of the entire Columbia River system, the use of a yet larger computer would be recommended. Even such problems, however, could probably be solved by use of a medium-sized computer facility.

The method proposed in this paper is one which is generally adaptable for many different systems and types of exact solutions. More extensive work and further experience with the problems of hydro storage, coupled with the use of the IBM 650 computer, will no doubt permit further improvement and extension of the application.

PUBLICATIONS

of the

ENGINEERING EXPERIMENT STATION

UNIVERSITY OF WASHINGTON

Requests for copies of publications should be addressed to the Director, Engineering Experiment Station, University of Washington, Seattle 5.

BULLETINS

48. The Influence of Pipe Thickness on Re-Entrant Intake Losses. Charles William Harris, 35 pp. 1928.
54. Elimination of Hydraulic Eddy Current Loss at Intake. Charles W. Harris. 30 pp. 1930.
55. Vacuum-Tube Control for Electric Wind-Tunnel Balances. Fred S. Eastman. 20 pp. 1931.
56. Constant Flow Characteristic of the Plane Orifice in Proximity to Side Walls. Charles William Haris. 18 pp. 1931.
58. Preliminary Tests of Thirteen Washington Coals in a Powdered Coal Boiler Plant at the University of Washington. George Samuel Wilson, Harry Fagan Yancey, and Joseph Daniels. 27 pp. 1931.
60. An Electromagnetic Balance for Force Measurement of Current Control. Fred Scoville Eastman. 29 pp. 1932.
61. Hydraulic Flow Characteristics of a Square-Edged Intake. Charles William Harris. 21 pp. 1932.
67. The Doherty Washer Cooler. Sybren Ruurd Tymstra. 61 pp. 1932.
69. Studies in Hydrogenation of Certain American Coals. Warren L. Beuschlein, Bert E. Christensen, and Calvert C. Wright. 24 pp. 1933.
72. The Derivation of the Mean Temperature Difference Between a Saturated Gas and a Cooling Liquid, and also the Derivation of the Mean Velocity of Such Gas. Sybren Ruurd Tymstra. 19 pp. 1933.
74. The Deformation of Hard Grade Reinforcing Steel During and After Construction in a Reinforced Concrete Building. Sergius Ivan Sergev. 33 pp. 1933.
75. The Water Tube Gas Condenser. Sybren Ruurd Tymstra. 24 pp. 1934.
77. A Study of Fundamental Relations of the Mathematical Theory of Elasticity. Arthur Melvin Winslow. 25 pp. 1934.
81. Influence of Two Secondary Factors in Weir Measurement. Charles William Harris. 16 pp. 1935.
84. Equations for Calculating Three Phase Symmetrical Components. Gordon Russell Shuck. 32 pp. 1935.
85. Discharge of Three Commercial Cippoletti Weirs. Robert Bowman Van Horn. 32 pp. 1935.
87. Open Water Testing of Ship Models. Thomas McKie Rowlands. 20 pp. 1935.
88. The Properties of Quartz Sands Washed from Kaolins of the Pacific Northwest. Hewitt Wilson and Frank Joseph Zvanut. 42 pp. 1936.
90. Hydroelectric Power in Washington. Part IV, Regional Electric-Power Transmission. The Grid System. Carl Edward Magnusson. 51 pp. 1936.
91. The Analysis and Control of Landslides. Robert Graham Hennes. 57 pp. 1936.
92. Moment Equations. A Method of Analysis for Continuous Beams and Rigid Frames. Alfred Jensen. 88 pp. 1937.
93. Electric Power Markets in Washington. Part I, Electric Heating of Residences. Carl Edward Magnusson. 40 pp. 1937.
94. Evaporation and Surface Cooling. George Samuel Wilson and William Lyle Dudley. 18 pp. 1937.
96. Hydroelectric Power in Washington. Part VI, International Boundary Waters. Carl Edward Magnusson. 17 pp. 1937.
98. Geology and Its Relation to the Occurrence of Oil in Washington. Charles Edwin Weaver. 16 pp. 1938.
100. Modernization of a Transit System. Factors that Determine the Choice of a Vehicle. George Lisle Hoard. 36 pp. 1940.
101. Buckling Criteria and Design of Two-Story Columns. Sergius I. Sergev. 48 pp. 1940.
102. An Investigation of Prestressed Stays. T. M. Rowlands and F. C. Smith. 36 pp. 1941.
103. Bismuth Bridge Magnetic Flux Meter. George Sherman Smith. 48 pp. 1940.
104. The Telechronometer, Telephone Service Measurement in Units of Time. Lyall Baker Cochran. 68 pp. 1941.
106. The Shearing Strength of Cement Mortar. Frederick C. Smith and Robert Q. Brown. 32 pp. 1941.
109. The Number of Teeth in Contact as a Vibration Factor in Involute Cut Gears. Sybren Ruurd Tymstra. 16 pp. 1942.
110. Solutions for Torsional Vibration Frequencies by Energy Method. Arthur Melvin Winslow. 22 pp. 1943.
111. Stabilized Earth Blocks for Protective Construction. Robert G. Hennes. 24 pp. 1943.
112. Hydraulic Models. Charles W. Harris. 17 pp. 1944.
114. The Effect of Shearing Forces on the Deflection and Strength of Beams. Sergius I. Sergev. 20 pp. 1947.
116. Aerodynamic Stability of Suspension Bridges with Special Reference to the Tacoma Narrows Bridge. Part I, Investigations Prior to October, 1941. F. B. Farquharson. 103 pp. 1949.
116. Aerodynamic Stability of Suspension Bridges with Special Reference to the Tacoma Narrows Bridge. Part II, Mathematical Analyses. F. C. Smith and G. S. Vincent. 63 pp. 1950.
116. Aerodynamic Stability of Suspension Bridges with Special Reference to the Tacoma Narrows Bridge. Part III, The Investigation of Models of the Original Tacoma Narrows Bridge Under the Action of Wind. F. B. Farquharson. 125 pp. 1952.
116. Aerodynamic Stability of Suspension Bridges with Special Reference to the Tacoma Narrows Bridge. Part IV, The Investigation of Models of the New Tacoma Narrows Bridge Under the Action of Wind. F. B. Farquharson. 104 pp. 1954.
116. Aerodynamic Stability of Suspension Bridges with Special Reference to the Tacoma Narrows Bridge. Part V, Extended Studies: Logarithmic Decrement, Field Damping, Prototype Predictions, Four Other Bridges. G. S. Vincent. 104 pp. 1954.
118. Investigations in Aeronautics from the Department of Aeronautical Engineering. Edited by R. M. Rosenberg. 120 pp. 1951.
119. The Extremum Principles of the Mathematical Theory of Elasticity and Their Use in Stress Analysis. (The Walker-Ames lectures of the Spring Quarter, 1950.) W. Prager (Brown University). 30 pp. 1951.

REPORTS

2. Iron and Steel Manufacture in Washington, Oregon, California, and Utah. Joseph Daniels. 69 pp. 1929.
3. Coal in Washington. Joseph Daniels. 17 pp. 1934.
4. Water Resources of Washington. Richard Gaines Tyler. 61 pp. 1938.

TECHNICAL NOTES

3. The Theory of Fractional Parts Developed for Structural Beam-Column Members Preceded by an Elementary Discussion of Simple Beam and Column Design. Sybren Ruurd Tymstra. 32 pp. 1943.
4. Gaging Chart for Horizontal Cylindrical Tanks with Flat Heads or with Convex Heads. Sybren Ruurd Tymstra. 4 pp. 1944.
5. Gaging Chart for Cylindrical Tanks with Different Types of Heads. Sybren Ruurd Tymstra. 7 pp. 1945.
6. The Heat Pump for Residential Heating. G. S. Smith, H. Christensen, and A. Presson. 23 pp. 1948.